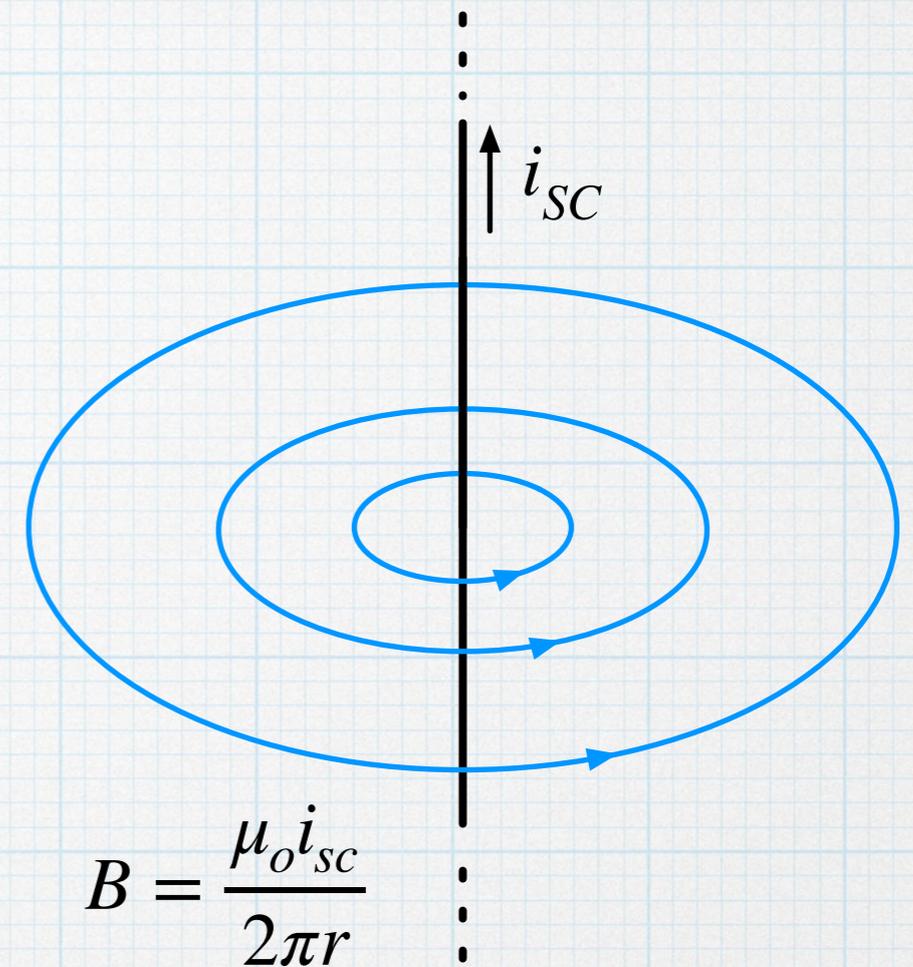# Inductance and inductors

Consider a short circuit that is carrying a current i_{SC} — we have seen shorts often and know that the no voltage developed, regardless of current level.

However, we also recall from basic electromagnetic that a current in a wire will have a magnetic field circulating around it. The field pattern is circular and the strength of the field diminishes with increasing distance from the wire.

i_{SC}

$$B = \frac{\mu_o i_{sc}}{2\pi r}$$

Whenever charges are moving in a conductor, there will be magnetic fields present. Since is takes some energy to generate the field, it can viewed as having some energy stored in it.

In a typical wire in a typical EE 201 circuit, the magnetic field and associated energy are very small and can probably be neglected in most instances.

Magnetic fields — a very short review

Magnetic fields are characterized by two different field quantities,

$H \rightarrow$ magnetic field strength, units of amperes /meter (A/m)

$B \rightarrow$ magnetic flux density, units of tesla (T) [$= \text{V·s}/\text{m}^2$ }
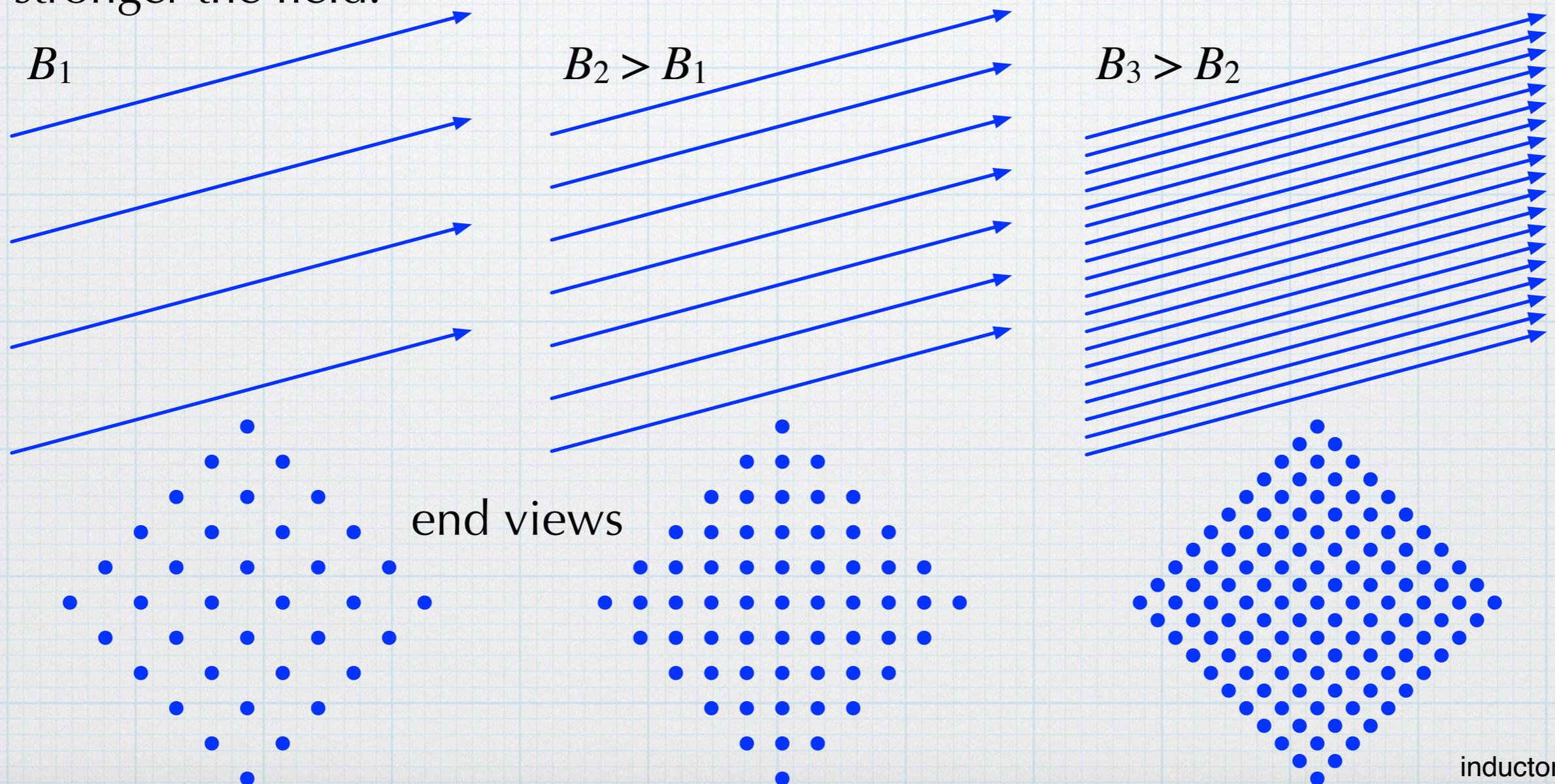
The two are related and in common materials, either can be used as the primary description of a magnetic field. In general, both are vector quantities. (We will ignore the vector nature of the magnetic fields in our simplified discussion of inductors.) The relationship between B and H is given by the *constitutive equation:*

$$B = \mu H$$

where μ is the *permeability* of the material where the magnetic field exists. The units of permeability are henries/meter (H/m). The free space value of permeability is $\mu_o = 4\pi \times 10^{-7}$ H/m. Most materials have the free-space value of permeability, but some materials — usually alloys that contain iron, nickel, cobalt, or gadolinium — have higher permeabilities. A higher permeability is characterized by a *relative permeability, μ_r,* which is the factor by which the permeability is increased over the free-space value.
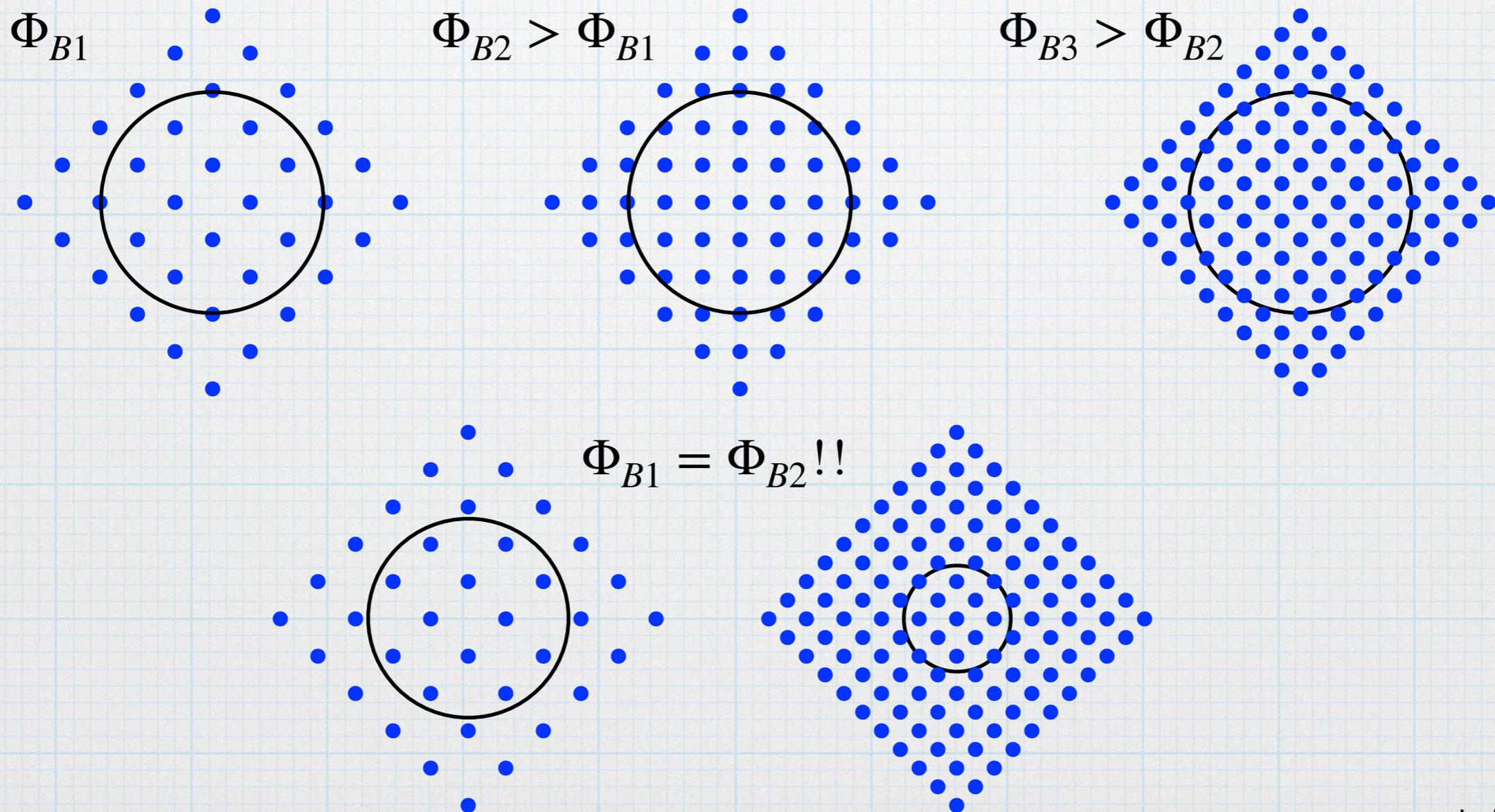
Magnetic flux density

In discussing inductors, we will focus on B, the magnetic flux density. Visualizing EM fields and making sense of the different quantities is always difficult. To develop a more intuitive feel for B, we can think about the field in terms of *field lines*. The magnetic flux density indicates how tightly packed the magnetic field lines are in a region of space. The more tightly packed they are (i.e. the higher the density), the stronger the field.

B_1

$B_2 > B_1$

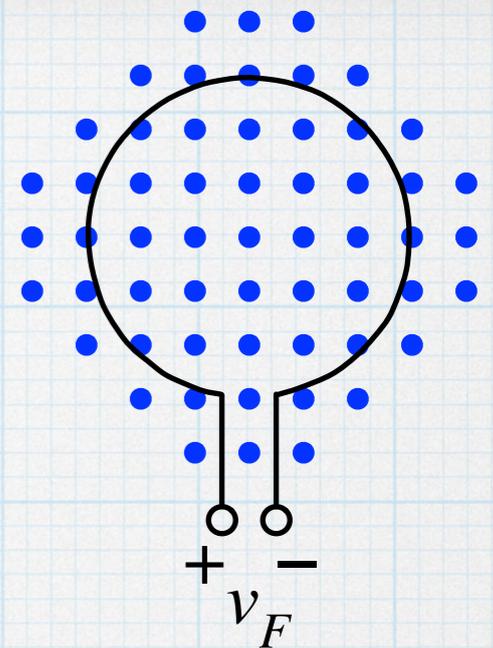$B_3 > B_2$

end views

Magnetic flux

Given a magnetic flux density, we can define an area in the space where the magnetic field exists. Within that area, there will be some number of magnetic field lines. The number within the area is called the magnetic flux.

$$\Phi_B = B \cdot A \qquad \text{Units of webers, Wb, } (= \text{V·s})$$

Φ_{B1}

$\Phi_{B2} > \Phi_{B1}$

$\Phi_{B3} > \Phi_{B2}$

$\Phi_{B1} = \Phi_{B2}!!$

Faraday's Law of Induction

Consider a magnetic field with magnetic flux density, B. A loop of wire defines an area A that has lines of magnetic field crossing through it. The loop defines a magnetic flux.

$$\Phi_B = B \cdot A$$

Faraday discovered that if a wire loop has magnetic field lines cutting through i — defining a magnetic flux in within the loop and if the magnetic flux is changing, there will be voltage a induced across the terminals of the wire. (More rigorously, an electric field is produced between the open ends of the wire. Of course, we know that if there is an electric field, there will be a corresponding voltage difference.)

$$v_F = -\frac{d\Phi_B}{dt}$$

The negative sign comes from "Lenz's Law", which says that the magnetic field created by the current in the loop caused by the induced v_F must be opposite the original magnetic field.

The induced voltage can be the result of the magnetic field itself changing (as with inductors — to be seen shortly) or by having the area change by moving the loop within the field (as in the case of electric generators). Or both.

Coil

Wrapping *N* turns of wire around a core creates a *coil*. If the turns of the coil are packed tightly together, the magnetic fields curling around each individual wire will merge to create an approximately uniform magnetic field extending down the length of the interior core. The interior magnetic flux density will be

$$B_{core} = \left(\frac{\mu \cdot N}{l} \right) i_{coil}$$

where *l* is the length of the core, μ is permeability of the core material and i_{coil} is the current in wire.

If the core has a cross-sectional area of *A*, the corresponding magnetic flux is

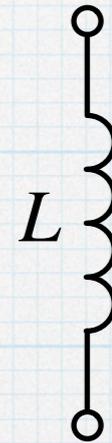$$\Phi_B = N \cdot B_{core} \cdot A = \left(\frac{\mu N^2 A}{l} \right) i_{coil}.$$

(The *N* turns increases the effective area intersecting the magnetic flux density.)

Inductance

$$\Phi_B = \left(\frac{\mu N^2 A}{l} \right) i_{coil}$$

The magnetic flux is directly proportional to the current. We can define the proportionality constant as the *self-inductance*.

$$\Phi_B = L \cdot i_{coil} \quad \text{where} \quad L = \frac{\mu N^2 A}{l}.$$

Inductor circuit symbol.

For a given amount of current, the field can be increased by: increasing the number of turns, increasing the diameter of the coil (increasing A), decreasing the length of the coil, or by having the core made of a material with higher permeability.

The units of inductance are henries or **H** (after American scientist Joseph Henry.) $1\,H = 1\,V \cdot s/A$. Typical values used in circuits range from about $1\,\mu H$ upwards to as much $0.1\,H$. (In power systems, much bigger inductors are possible.)

Inductor voltage

A straight-forward application of Faraday's Law to the magnetic flux of an inductor gives:

$$\Phi_B = L \cdot i_{coil}$$

$$-\frac{d\Phi_B}{dt} = v_L = L\frac{di_L}{dt}$$

leading to the all-important current-voltage relationship for an inductor:

$$\boxed{v_L = L\frac{di_L}{dt}}$$

A consideration of Lenz's Law leads to the indicated relationship between current direction and voltage polarity — identical to relationship in a resistor or capacitor.
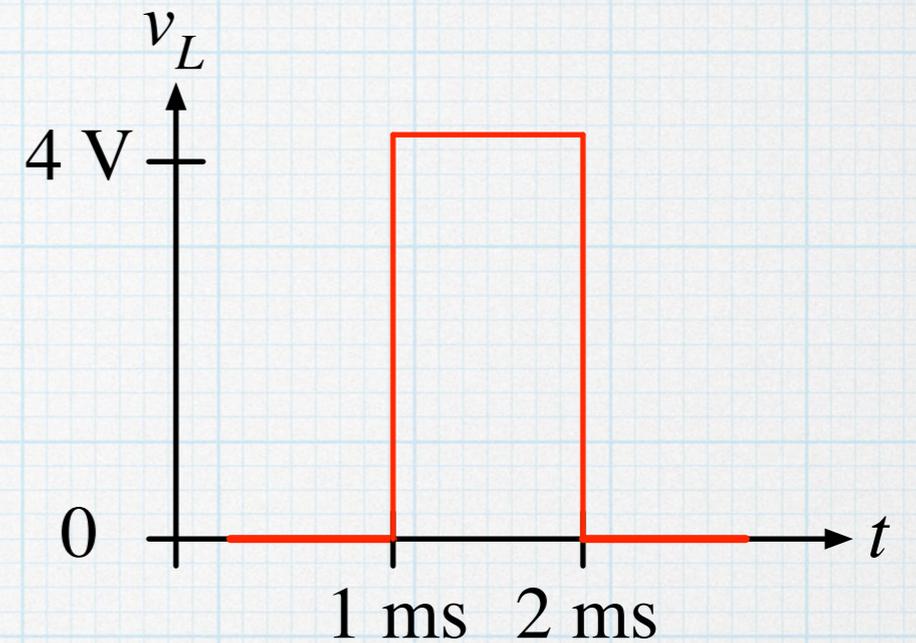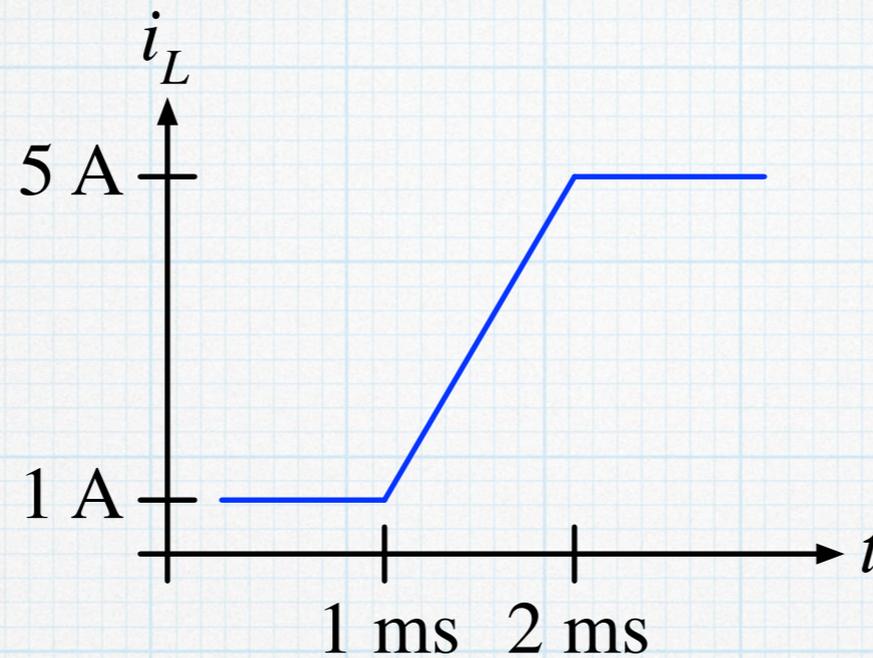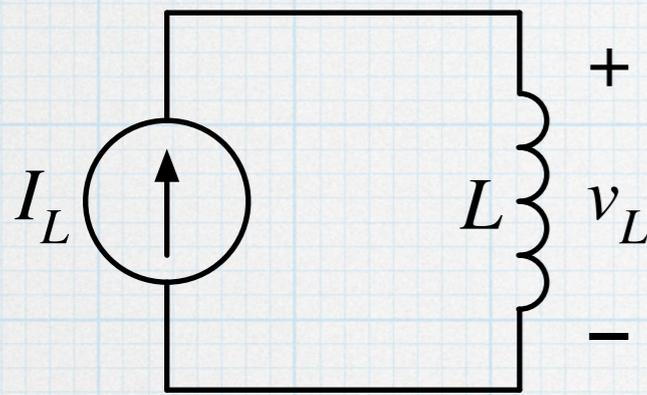
If the current is not changing, then there is no voltage and the inductor behaves like a simple (if somewhat lengthy) short circuit. When the current is changing, the changing field causes a voltage to be developed that is proportional to the time derivative of the current.

Conversely, if there is a voltage on the inductor, then current must be changing. The current can be calculated by turning the above equation around:

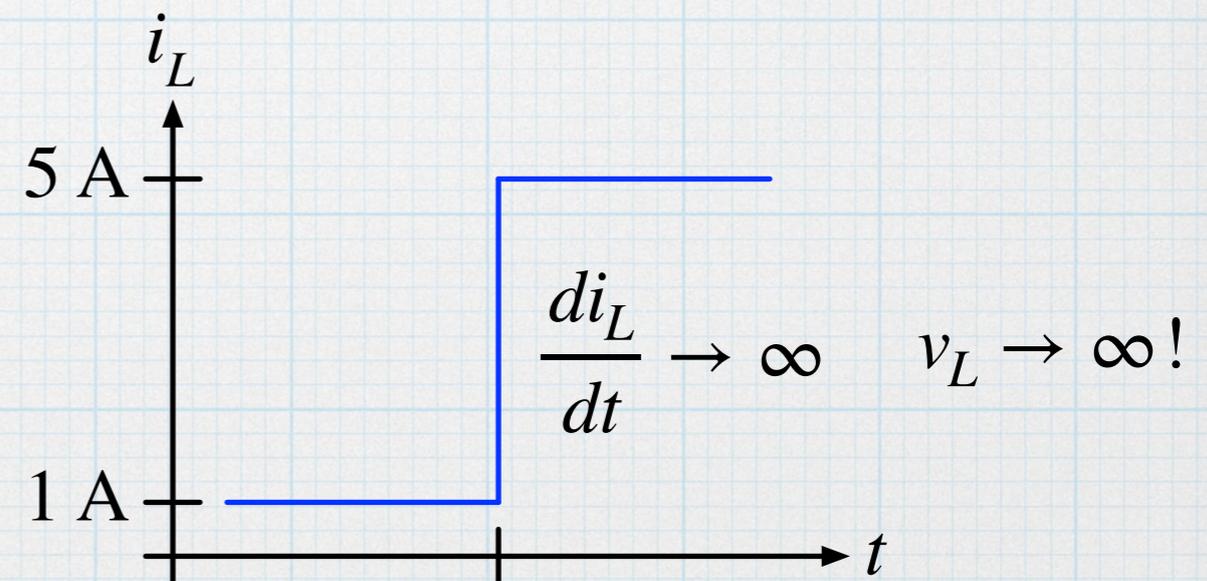$$i_L(t) - i_L(0) = \frac{1}{L}\int_0^t v_L(\tau)\,d\tau$$

Example

For $t < 1$ms, $di_L/dt = 0 \rightarrow v_L = 0$.

For 1 ms $< t < 2$ ms, $di_L/dt = 4000$ A/s $\rightarrow v_L = (0.001$ H$)(4000$ A/s$) = 4$ V.

For $t > 2$ ms, $di_L/dt = 0 \rightarrow v_L = 0$.

Due to the derivative in the inductor equation, the current cannot change instantaneously. This would require an infinitely large voltage and an infinitely large amount of power.

$$\frac{di_L}{dt} \rightarrow \infty \qquad v_L \rightarrow \infty \,!$$

However, the inductor *voltage* can change instantaneously, — the slope change abruptly, as long as the current is continuous.

Inductor energy

The induced magnetic field requires energy in order for it to build up — the magnetic field represents energy stored in the inductor.

To determine the stored energy, start with power. When the inductor is "amping up", the power is

$$P_L(t) = i_L(t) v_L(t) = Li_L \frac{di_L}{dt}$$

The change in energy due to a change in current can be found by integrating the power over time.

$$\Delta E = \int_{i_L(0)}^{i_L(t)} P_L(t)\, dt = \int_{i_L(0)}^{i_L(t)} i_L di_L = \frac{1}{2} L \left[i_L^2(t) - i_L^2(0) \right]$$
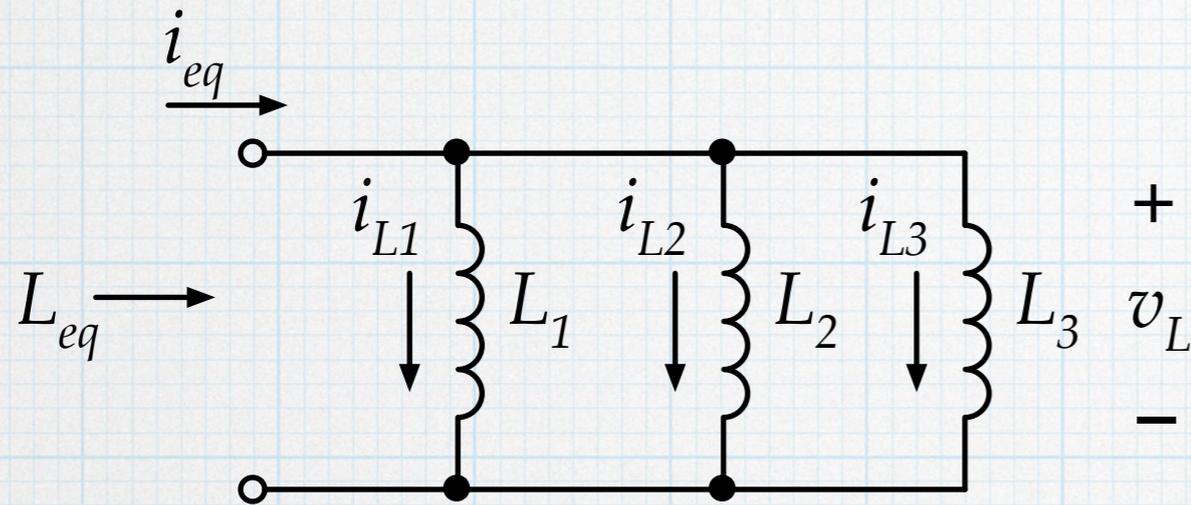
If we choose $i_L(0) = 0$, which corresponds to no stored magnetic energy, the inductor energy corresponding to given a current can written as

$$E_L = \frac{1}{2} Li_L^2$$

Typical value: For $L = 1$ mH and $i_L = 10$ A, $E = 0.05$ J — not a whole lot.

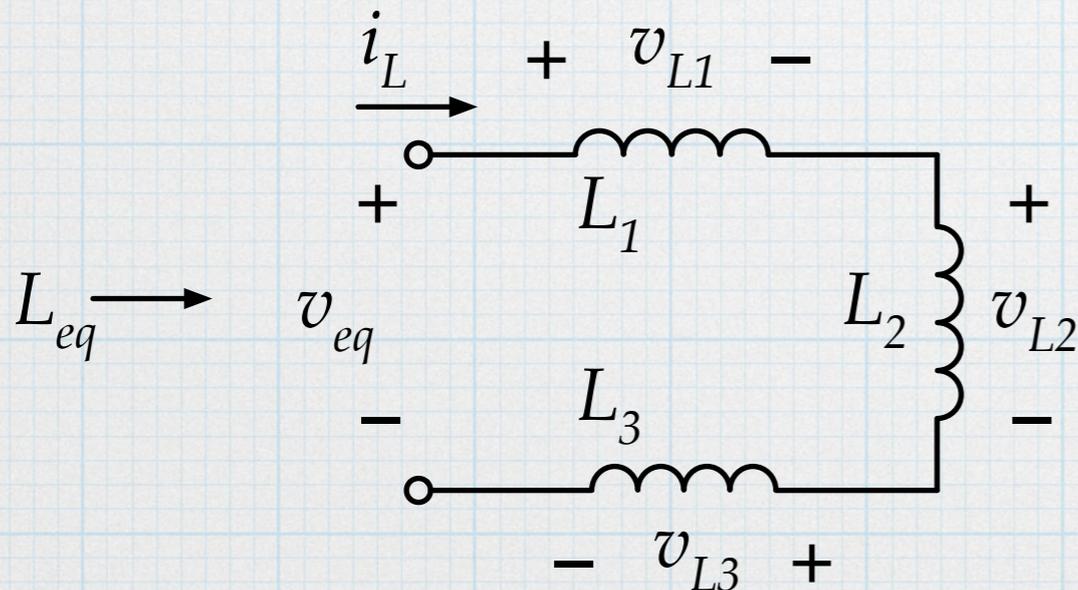# Series and parallel inductor combinations

Parallel

$$i_s = i_1 + i_2 + i_3$$

$$i_{eq} = i_{L1} + i_{L2} + i_{L3}$$

$$\frac{di_{eq}}{dt} = \frac{di_{L1}}{dt} + \frac{di_{L2}}{dt} + \frac{di_{L3}}{dt}$$

$$\frac{1}{L_{eq}} = \frac{1}{L_{L1}} + \frac{1}{L_{L2}} + \frac{1}{L_{L3}}$$

$$\frac{v_L}{L_{eq}} = \frac{v_L}{L_{L1}} + \frac{v_L}{L_{L2}} + \frac{v_L}{L_{L3}}$$

Series

$$v_{eq} = v_{L1} + v_{L2} + v_{L3}$$

$$L_{eq}\frac{i_L}{dt} = L_1\frac{i_L}{dt} + L_2\frac{i_L}{dt} + L_3\frac{i_L}{dt}$$

$$= (L_1 + L_2 + L_3)\frac{i_L}{dt}$$

$$L_{eq} = L_1 + L_2 + L_3$$

Inductor combinations are identical to those for resistors.