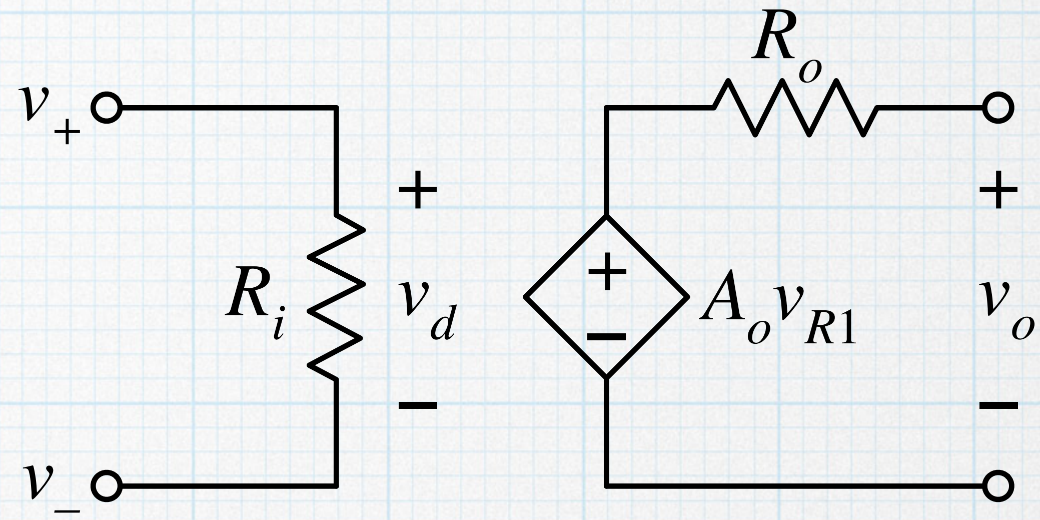
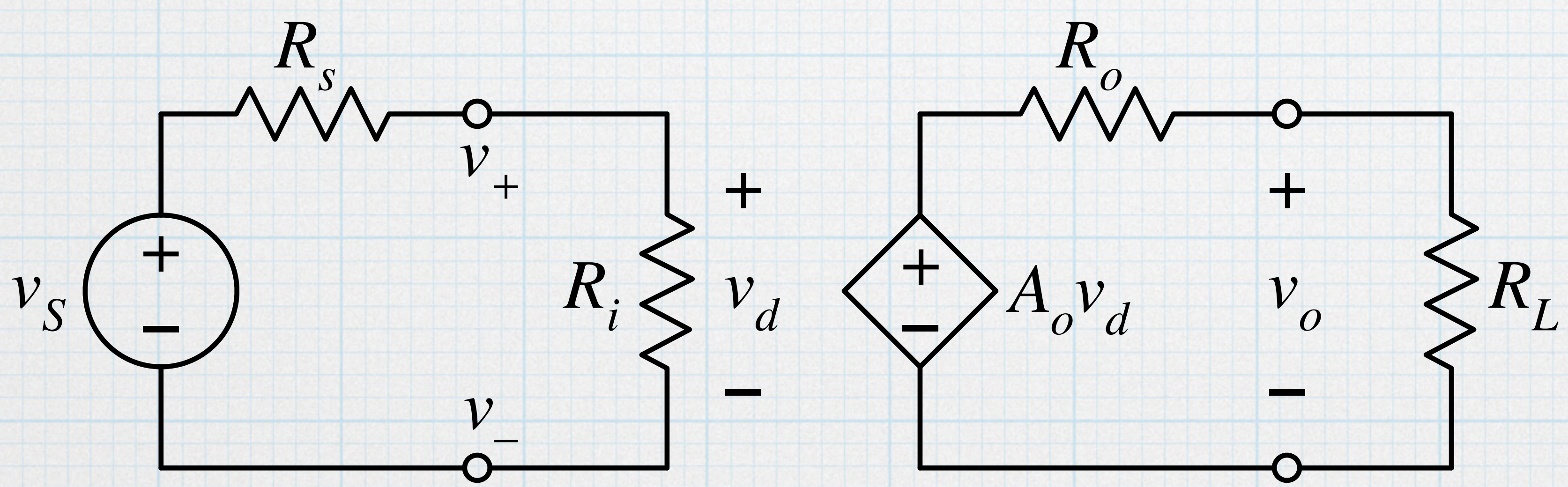
Operational amplifiers (Op amps)

Recall the basic two-port model for an amplifier. It has three components: input resistance, R_i, output resistance, R_o, and the open-loop voltage gain, A_o.



It is essential to specify all three parts of the model — when we attach an input source and an output load, the voltage developed across the load depends very much on R_i and R_o, as well as A_o.

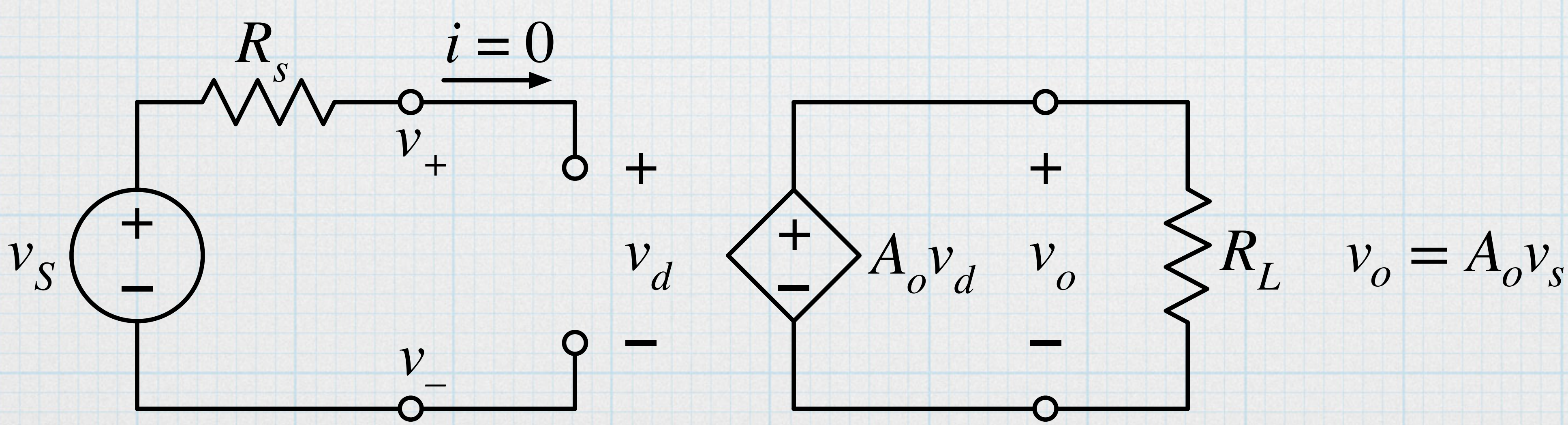


$$v_o = \left[\frac{R_i}{R_i + R_S} A_o \frac{R_L}{R_L + R_o} \right] v_s$$

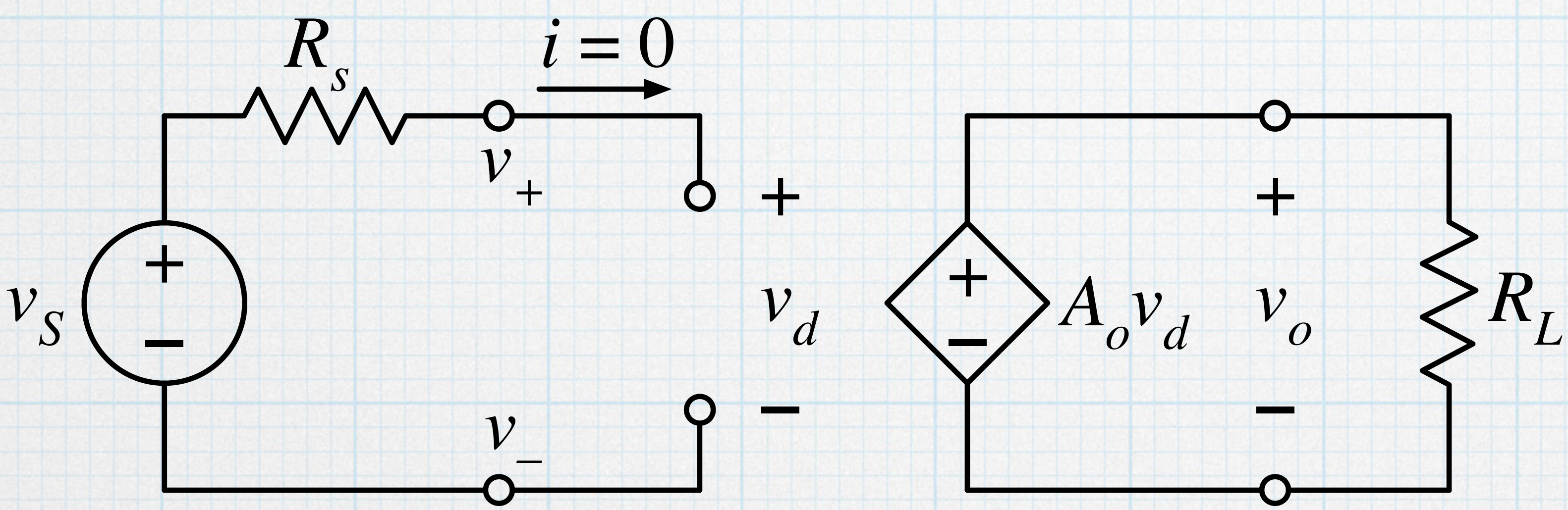
To minimize the effects of the input and output voltage dividers, we might try to make the input resistance be as big as possible — in the limit it would be open circuit — and the output resistance be as small as possible — in the limit it would be a short circuit.

$$v_o = \left[\frac{R_i}{R_i + R_S} A_o \frac{R_L}{R_L + R_o} \right] v_s \xrightarrow{R_i \to \infty\,,\, R_o \to 0} v_o = A_o v_s$$

Operational amplifiers (op amps, for short) are a class of amps that come close to meeting these input and output resistance extremes. We can define an *ideal op amp* as one having truly infinite input resistance and zero output resistance. No real op amp meets these specifications, but typical devices come close enough that we can use the ideal as reasonable starting point.



Op amps have a third distinguishing feature — a very large value for A_o. In the ideal case, the open-loop gain is assumed to be infinitely large! This would seem to cause some conceptual (and practical) difficulties, since then if $v_d = v_S$ had any finite value, the output would go to infinity. The only way to keep the output from “blowing up” would be to have $v_S = 0$. This all seems a little odd.



If $A_o \rightarrow \infty$ then $v_o \rightarrow \infty$.

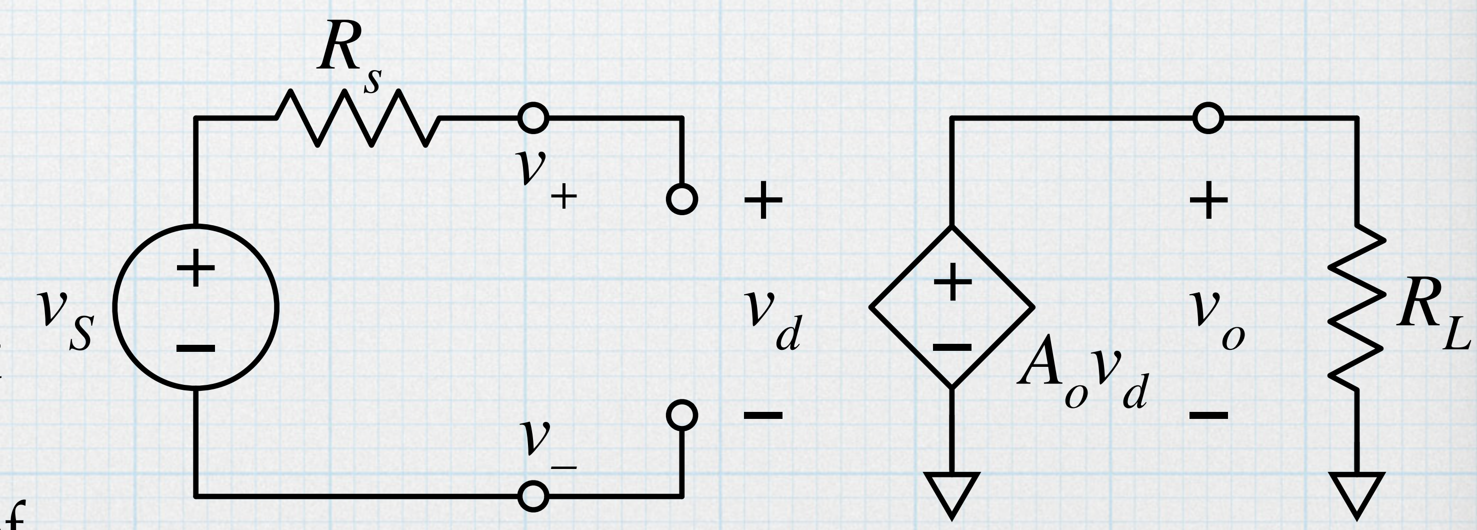
Before we see how to handle the business of infinite gain, let’s list the properties of an ideal op amp.

1. Infinite input resistance, $R_i \rightarrow \infty$. (An open circuit.)
2. Zero output resistance, $R_o = 0$. (A short circuit.)
3. Infinite open-loop gain, $A_o \rightarrow \infty$.

Ground in electronic circuits

In contrast to earlier circuits that we have analyzed, electronic circuits always have a specified node defined to be ground. Before now, the only time we invoked a ground node was when using the node-voltage analysis. Then we would *choose* a node to be ground ($v = 0$) in order to reduce the number of unknowns. An electronic circuit always requires one or more DC power supplies in order to energize the circuits in the chips. The presence of the power supply allows us to define one connection of the supply to be the ground for the entire circuit. We will discuss the role of the power supplies later, but the idea of a single, pre-defined ground does have an immediate effect our op amp model. The output voltage of most (but not all) op amps is defined with respect to the ground node rather than as the difference between two isolated output leads.

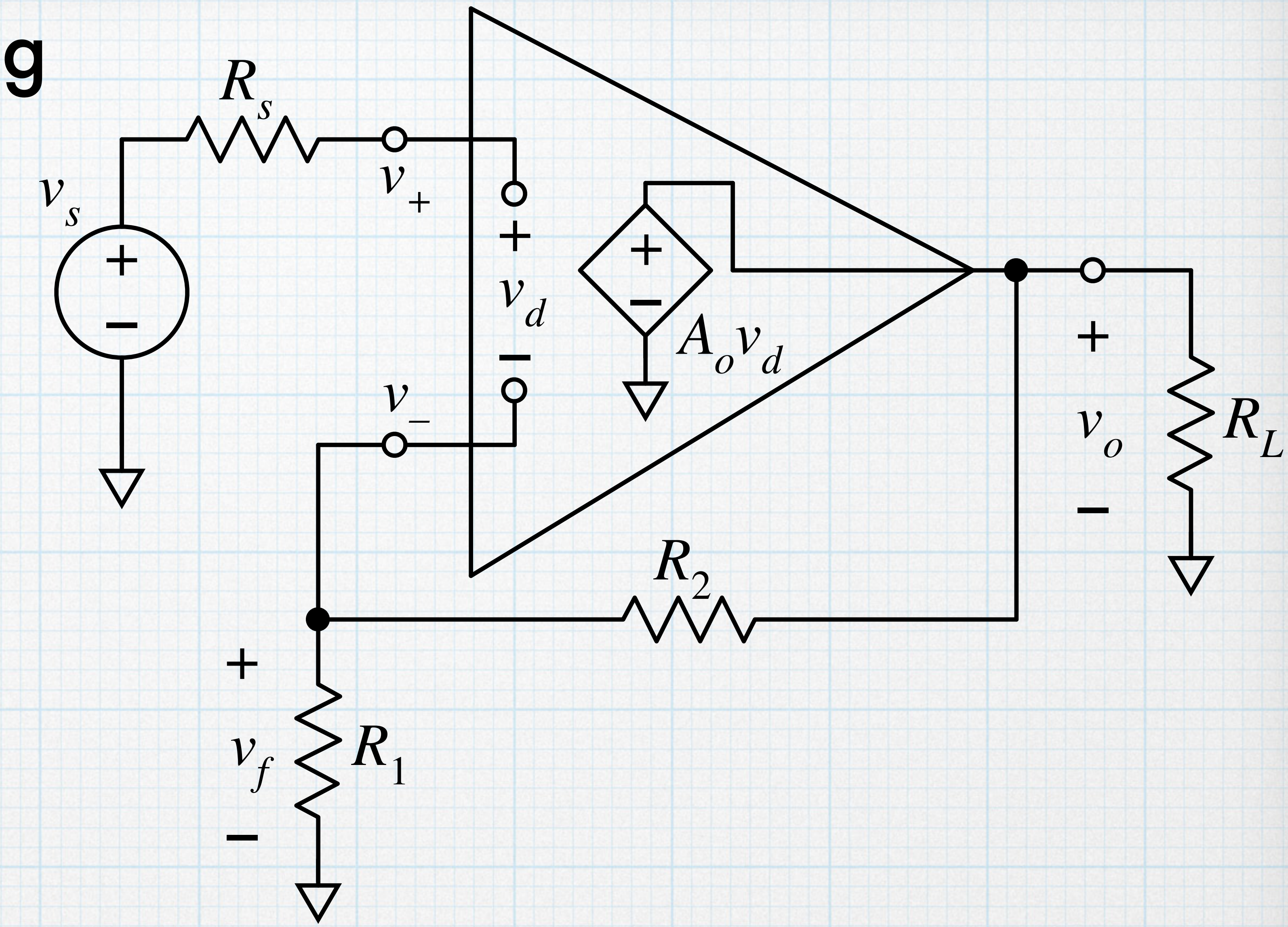
Most op amps have this "differential input, single-ended output" configuration, although there are some differential output versions. Using the defined ground changes the output side of the model as shown.



Single-ended output.

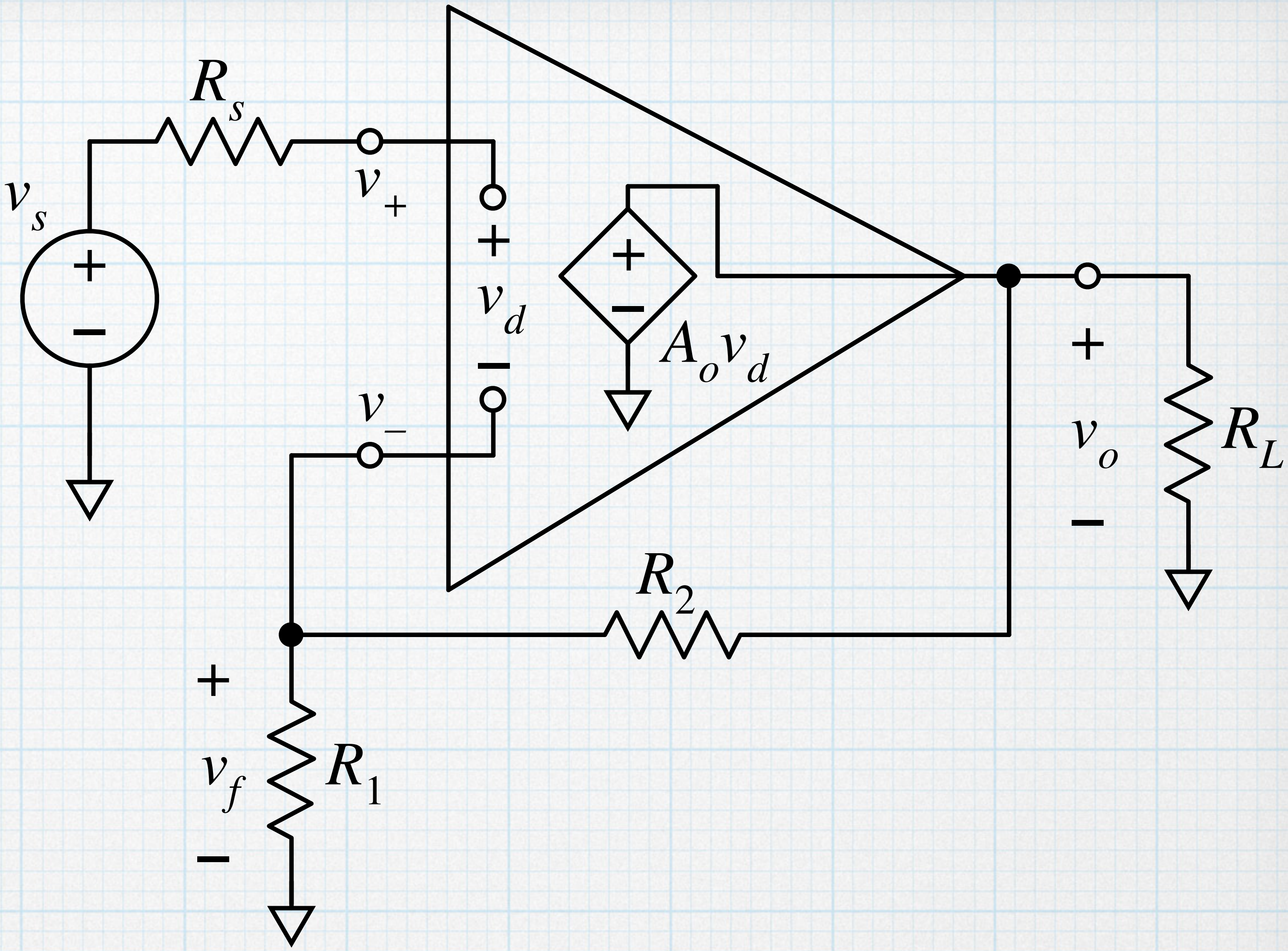
Feedback – non-inverting

Now we are ready to tame the infinite-gain issue. Of course, the trick is to use feedback. In fact, we have already done it — refer back to the feedback notes. In the circuit at right, known as a non-inverting amplifier, we have used a standard voltage-divider feedback loop together with the "almost ideal" op amp.



In drawing the circuit, we have made use of two bits of standard schematic nomenclature — the triangle symbol for the amplifier and the copious use of ground symbols. All the points with ground symbols form a single node. We could have connected all the ground points with wires, but the resulting schematic would have been almost un-intelligible. And this is a very simple electronic circuit. In more involved applications (EE 230), the schematics will have ground symbols all over the place.

Finally, we can get to the circuit analysis. First we note that R_S and R_L are essentially irrelevant. Since no current flows into the positive input terminal, there is no voltage drop across R_S and $v_+ = v_s$. Since R_L is connected directly to the dependent source, $v_o = A_o v_d$, no matter what value R_L may have.



$$v_o = A_o v_d$$

$$i_{R2} = i_{R1} \;\rightarrow\; \frac{v_o - v_f}{R_1} = \frac{v_f}{R_2} \;\rightarrow\; v_f = \frac{R_1}{R_1 + R_2} v_o$$

(Duh. It's a voltage divider.)

$$v_d = v_+ - v_f = v_s - v_f$$

Put it all together to write v_o in terms of v_s.

$$v_o = \left[\frac{A_o}{1 + A_o\left(\frac{R_1}{R_1 + R_2}\right)} \right] v_s$$

It looks like a classic feedback result.

$$v_o = \left[\frac{A_o}{1 + A_o\left(\frac{R_1}{R_1 + R_2}\right)}\right] v_s$$

Re-arranging slightly.

$$v_o = \left[\frac{\frac{R_2 + R_1}{R_1}}{1 + \frac{1}{A_o}\left(\frac{R_2 + R_1}{R_1}\right)}\right] = G_o v_s$$ G_o is the closed-loop gain.

If A_o is really big (approaching infinity)

$$A_o \to \infty\,, G_o \to \frac{R_2 + R_1}{R_2} = 1 + \frac{R_2}{R_1}$$

Even though A_o is huge, it has essentially disappeared from the final result. This is just a repeat of the feedback results seen earlier.

We note again a key aspect of feedback by calculating v_-:

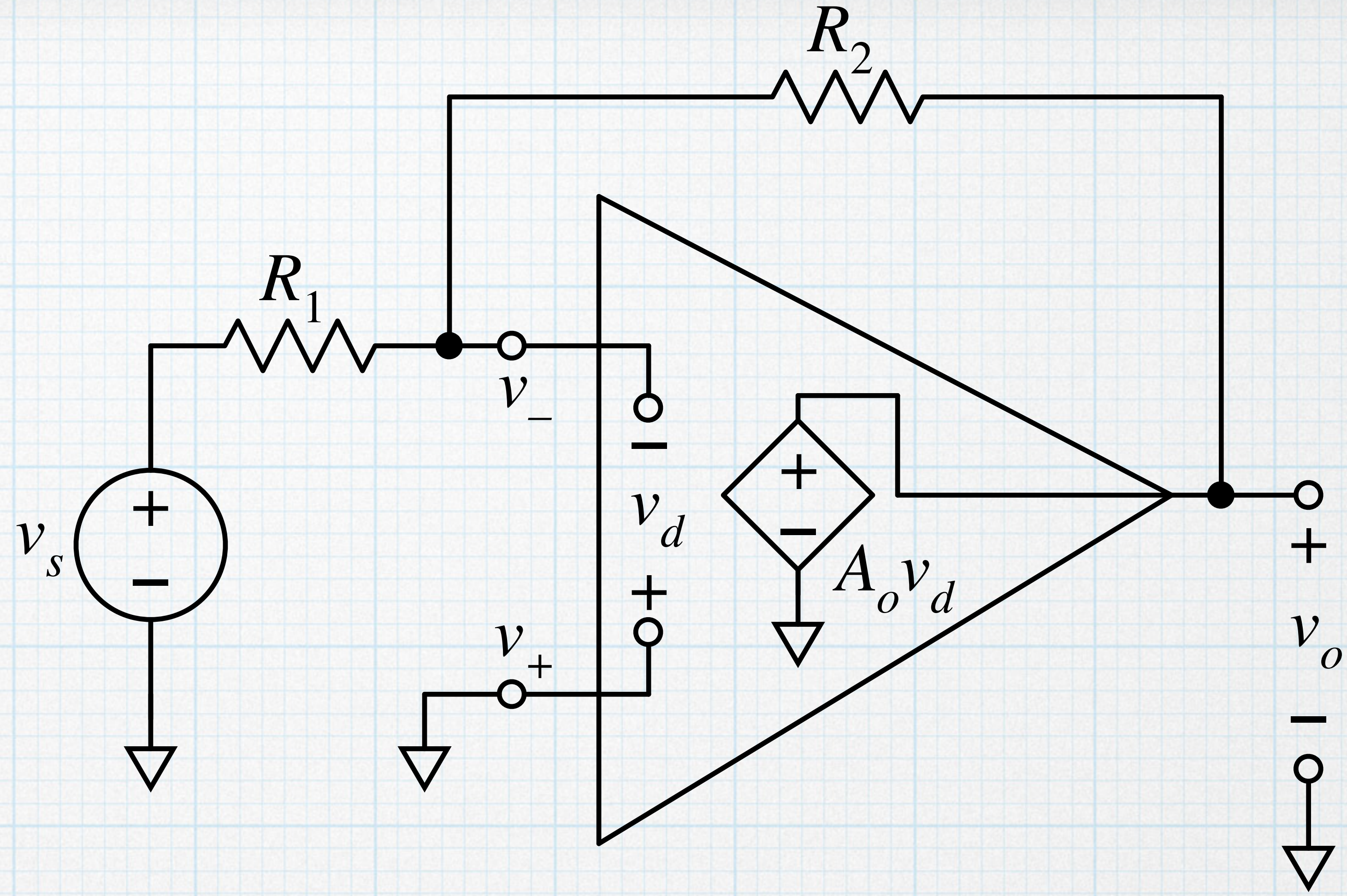
$$v_- = v_f = \frac{R_1}{R_1 + R_2} v_o = \left(\frac{R_1}{R_1 + R_2}\right)\left(\frac{R_1 + R_2}{R_1}\right) v_s = v_s = v_+$$

The feedback forces the difference signal to be zero: $v_d = v_+ - v_- = 0$.

The negative feedback has forced the negative input to have the same voltage as the positive input.

Feedback – inverting

Here is another amplifier circuit using feedback. The feedback network is less obvious, but it is negative feedback since the output is tied back to the negative input. A bit of circuit analysis leads to the closed-loop gain.



$$v_o = A_o v_d = -A_o v_-$$

$$i_{R1} = i_{R2} \;\rightarrow\; \frac{v_s - v_-}{R_1} = \frac{v_- - v_o}{R_2} \;\rightarrow\; v_o = -\frac{R_2}{R_1}v_s + \left(1 + \frac{R_2}{R_1}\right)v_-$$

Inserting $v_- = -v_o/A_o$ and re-arranging,

$$v_o = \left[\frac{-\frac{R_2}{R_1}}{1 + \frac{1}{A_o}\left(\frac{R_2 + R_1}{R_1}\right)}\right] = G_o v_s$$

G_o is the closed-loop gain.

$$v_o = \left[\frac{-\frac{R_2}{R_1}}{1 + \frac{1}{A_o}\left(\frac{R_2 + R_1}{R_1}\right)} \right] = G_o v_s$$

If A_o is really big

$$A_o \to \infty, G_o \to -\frac{R_2}{R_1}$$

We can calculate v_-:

$$v_- = \frac{v_o + \frac{R_2}{R_1} v_s}{1 + \frac{R_2}{R_2}} = 0$$

Once again, we see that the feedback forces the inputs to have the same voltage. In this case, $v_- = v_+ = 0$. This is interesting — even though v_- is not directly connected to ground, the perfect feedback has caused it to have the same value as v_+, which is connected to ground. This is known as a *virtual ground*.

The op-amp "rules".

Our observation that $v_- = v_+$ in these two examples suggests a more efficient means for analyzing op amp circuits. Instead of using the two-port model with a gain of A_o, finding an expression for closed-loop gain that includes A_o, and then letting $A_o \to \infty$, we can "skip the middleman" and assume at the outset that negative feedback will force $v_- = v_+$. Then we can calculate the ideal gain expression directly. Now we have two simple rules to use for analyzing ideal op amps:

1. $i_+ = i_- = 0$
2. $v_- = v_+$, when there is a negative feedback loop.

Each "rule" derives from one aspect of the ideal op amp

1. $R_i \to \infty$: no input currents.
2. $A_o \to \infty$: perfect feedback, $v_- = v_+$

The third aspect of the ideal op amp, $R_o = 0$, will come into play when we connect a load to the output or made make a cascaded arrangement where one op amp circuit follows another.

Non-inverting amp — redux

Now let's re-examine the non-inverting and inverting feedback amplifier configurations using the op amp rules.

Note that $v_+ = v_s$. Write a node-voltage equation at the inverting input terminal.

$$\frac{v_o - v_-}{R_2} = \frac{v_-}{R_1} + i_-$$

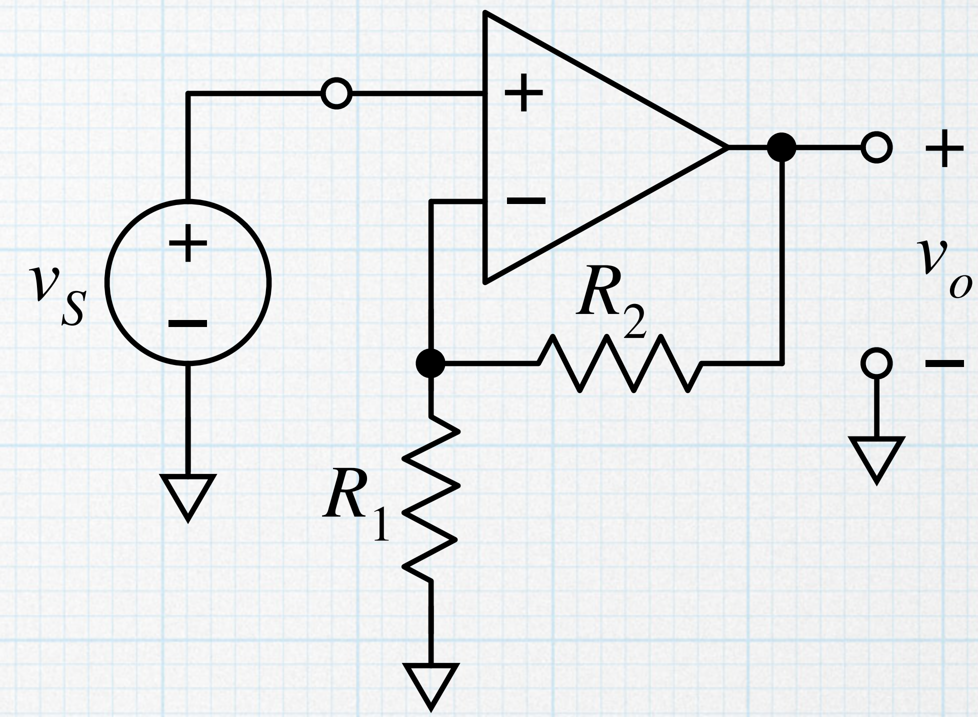
Apply the op amp rules:
$i_- = 0$ and $v_- = v_+ = v_s$.

$$\frac{v_o - v_s}{R_2} = \frac{v_s}{R_1}$$

$$v_o = \left(1 + \frac{R_2}{R_1}\right) v_s$$

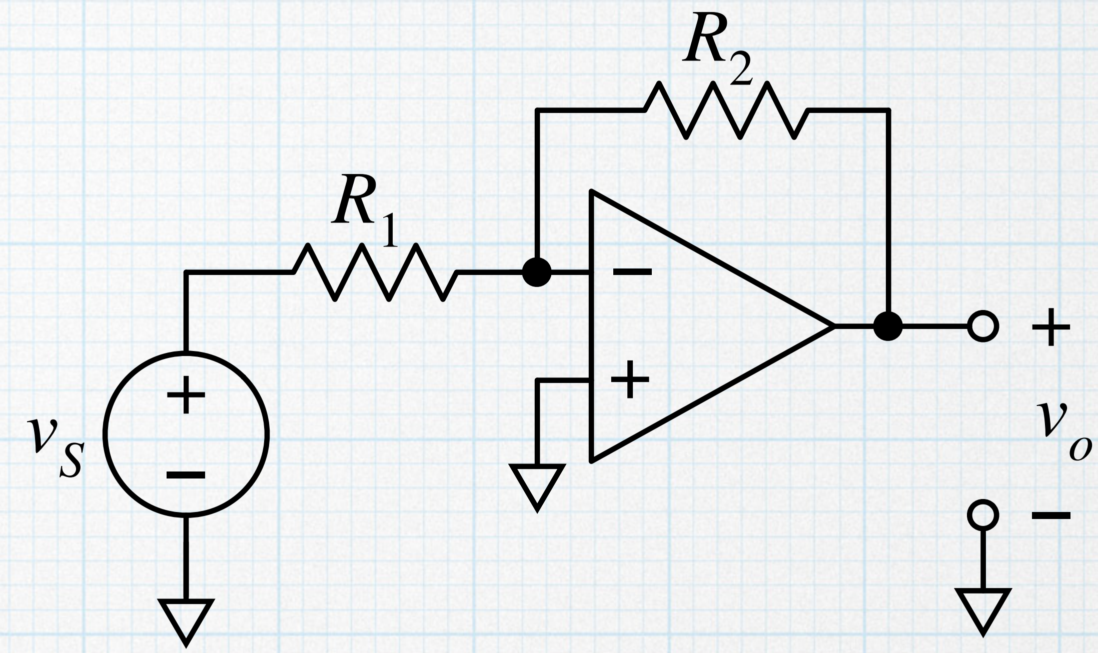
$$G_o = \frac{v_o}{v_S} = 1 + \frac{R_2}{R_1}$$

So simple!



Since we are not explicitly including A_o in the analysis, we can use a more compact symbol for the op amp. This is the standard — i.e. most typical — amp symbol.

Inverting amp – redux



Same basic approach. Note that $v_+ = 0$.

Write a node-voltage equation at the inverting input terminal.

$$\frac{v_s - v_-}{R_1} = \frac{v_- v_o}{R_2} + i_-$$

Apply the op amp rules: $i_- = 0$ and $v_- = v_+ = 0$ (virtual ground)

$$\frac{v_s}{R_1} = \frac{-v_o}{R_2}$$

$$v_o = \left(-\frac{R_2}{R_1}\right) v_s$$

$$G_o = \frac{v_o}{v_s} = -\frac{R_2}{R_1}$$

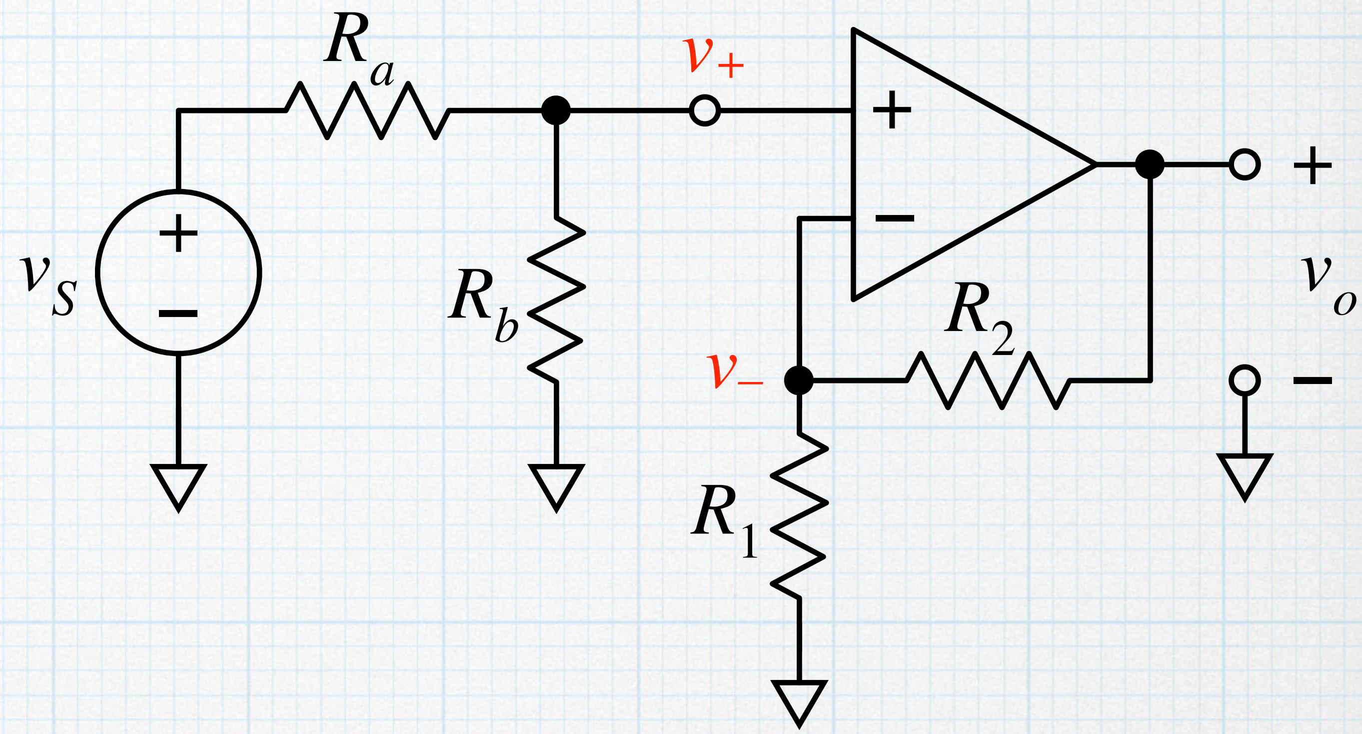
The negative sign is what make this “inverting”.

General approach for solving circuits with ideal op amps

In principle, any of the 201 circuit analysis techniques are viable when solving op-amps.

- Because of the ground defined by the power supplies, the node-voltage technique is almost always useful.
- Mesh currents might be applicable, but the fact that no current flows in the input leads can making finding useful meshes tricky. (Save mesh current for EE 303.)
- With multiple inputs, superposition may be useful.
- Voltage dividers are always useful. Because no current flows in the input leads, we can apply voltage dividers directly to the inputs.
- Usually, writing a node-voltage equation at the output is not useful, because we can't know the output current.
- Be careful with the idea of the virtual short (or virtual ground). It is NOT a true short circuit in regards to currents.
- Generally, write some NV equations at the inputs, set $v_+ = v_-$ and see what happens.

Example 1 – non-inverting



Hmmm. Try writing node-voltage equations at each of the inputs.

$$\frac{v_o - v_-}{R_2} = \frac{v_-}{R_1} + i_-$$

$$\frac{v_s - v_+}{R_a} = \frac{v_+}{R_b} + i_+$$

First, apply $i_+ = i_- = 0$:

$$\frac{v_o - v_-}{R_2} = \frac{v_-}{R_1} \longrightarrow v_o = \left(1 + \frac{R_2}{R_1}\right) v_-$$

Non-inverting amp result.

$$\frac{v_s - v_+}{R_a} = \frac{v_+}{R_b} \longrightarrow v_+ = \left(\frac{R_b}{R_a + R_b}\right) v_s$$

A simple voltage divider.

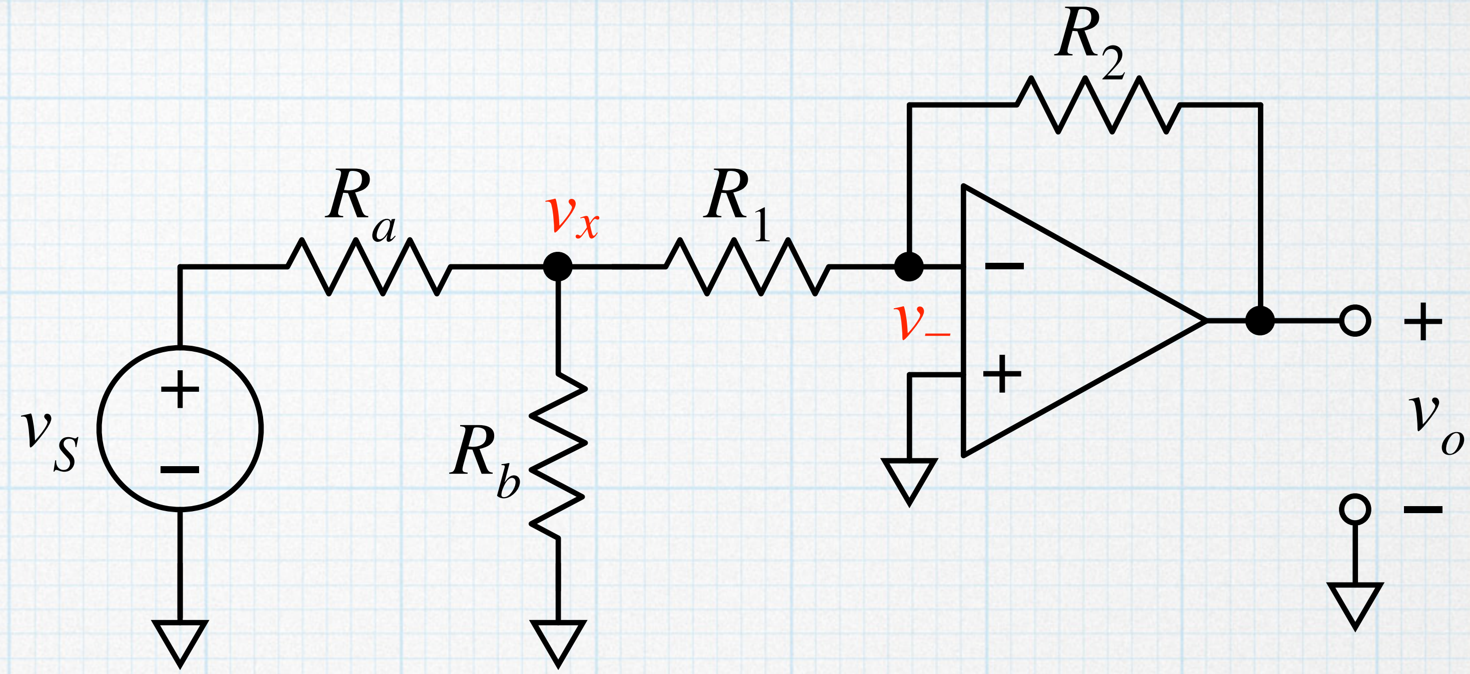
Next $v_+ = v_-$:

$$v_o = \left(1 + \frac{R_2}{R_1}\right) v_- = \left(1 + \frac{R_2}{R_1}\right) v_+ = \left(\frac{R_b}{R_a + R_b}\right)\left(1 + \frac{R_2}{R_1}\right) v_s$$

The product of two simple results.

Example 2 – inverting

How about this? Will it be the product of a voltage divider and an inverting gain? We should proceed carefully — write two node voltage equations.



$$\frac{v_s - v_x}{R_a} = \frac{v_x}{R_b} + \frac{v_x - v_-}{R_1} \quad \text{and} \quad \frac{v_x - v_-}{R_1} = \frac{v_- - v_o}{R_2} + i_-$$

Apply the op amp rules: $i_- = 0$ and $v_- = v_+ = 0$ (virtual ground)

$$\frac{v_s - v_x}{R_a} = \frac{v_x}{R_b} + \frac{v_x}{R_1} \longrightarrow v_x = \frac{v_s}{1 + \frac{R_a}{R_b} + \frac{R_a}{R_1}}$$

A voltage divider, of sorts

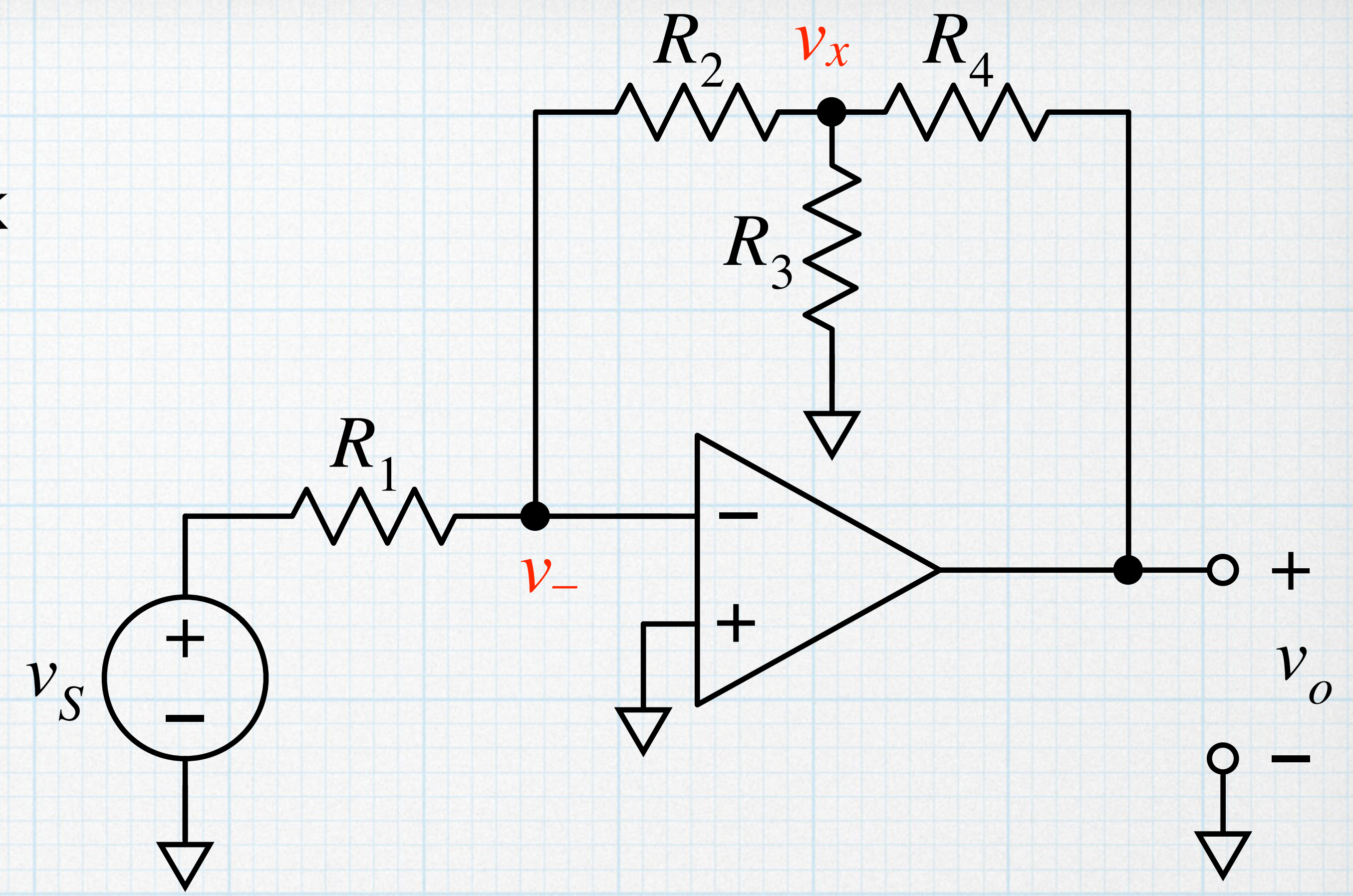
$$\frac{v_x}{R_1} = \frac{-v_o}{R_2} \longrightarrow v_o = -\left(\frac{R_2}{R_1}\right) v_x$$

Inverting amp.

Put the two results together: $v_o = \left(\dfrac{-\frac{R_2}{R_1}}{1 + \frac{R_a}{R_b} + \frac{R_a}{R_1}} \right) v_s$

Not difficult, but not what we might have guessed at the outset.

Example 3 – inverting



Wow — messing with the feedback path. Maybe this is impossible to solve? Nope. It's quite easy — just apply the rules. Start with two node-voltage equations:

$$\frac{v_s - v_-}{R_1} = \frac{v_- - v_x}{R_2} + i_-$$

$$\frac{v_- - v_x}{R_2} = \frac{v_x}{R_3} + \frac{v_x - v_o}{R_4}$$

Same rules as the previous example : $i_- = 0$ and $v_- = v_+ = 0$.

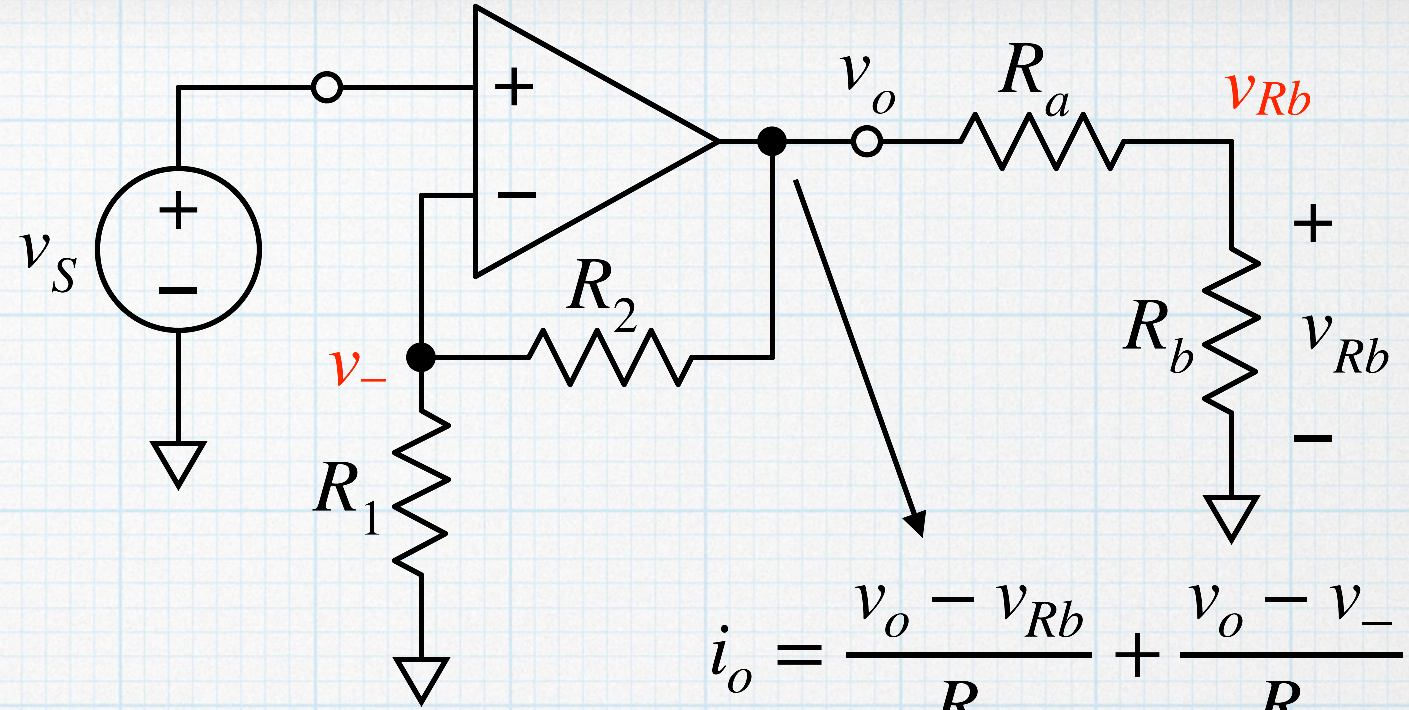
$$\frac{v_s}{R_1} = \frac{-v_x}{R_2} \longrightarrow v_x = -\left(\frac{R_2}{R_1}\right) v_s$$

$$\frac{-v_x}{R_2} = \frac{v_x}{R_3} + \frac{v_x - v_o}{R_4} \longrightarrow v_o = \left(1 + \frac{R_4}{R_2} + \frac{R_4}{R_3}\right) v_x$$

Put it together: $$v_o = -\left(1 + \frac{R_4}{R_2} + \frac{R_4}{R_3}\right)\left(\frac{R_2}{R_1}\right) v_s$$ Piece of cake.

Example 4 – non-inverting

Non-inverting amp with something attached to the output. Find v_{Rb} in team of v_s. A couple of node-voltage equations gives:



$$i_o = \frac{v_o - v_{Rb}}{R_a} + \frac{v_o - v_-}{R_2}$$

This NV equation is not useful! We don't know i_o.

$$\frac{v_o - v_-}{R_2} = \frac{v_-}{R_1} + i_- \quad \text{and} \quad \frac{v_o - v_{Rb}}{R_a} = \frac{v_{Rb}}{R_b}$$

Apply the op amp rules to the left equation: $i_- = 0$ and $v_- = v_+ = v_s$.

$$\frac{v_o - v_s}{R_2} = \frac{v_s}{R_1} \longrightarrow v_o = \left(1 + \frac{R_2}{R_1}\right) v_s$$

Usual on-inverting amp.

$$\frac{v_o - v_{Rb}}{R_a} = \frac{v_{Rb}}{R_b} \longrightarrow v_{Rb} = \left(\frac{R_b}{R_a + R_b}\right) v_o$$

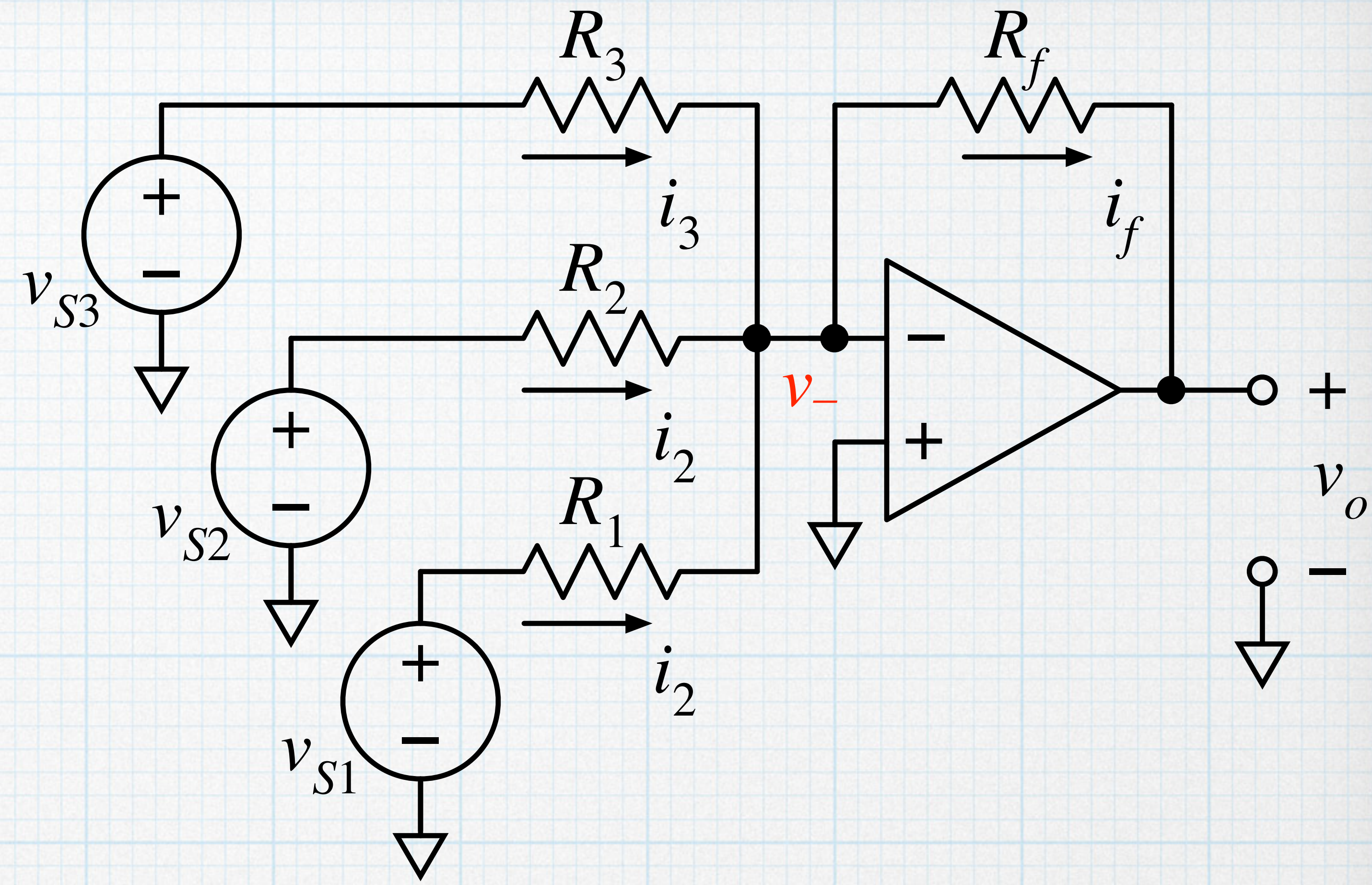
A simple voltage divider.

$$v_o = \left(\frac{R_b}{R_a + R_b}\right)\left(1 + \frac{R_2}{R_1}\right) v_s$$

Again, the product of two simple results. Result is identical to Example 1!

Summing amp

Something new — a summing amp. It looks a bit like an inverting amp, but with some extra inputs. Since there are multiple inputs, we cannot calculate a single gain for the circuit, be we can calculate v_o as a function of the various inputs.



Node voltage at inverting input:

$$i_{R1} + i_{R2} + i_{R3} = i_f + i_- \;\rightarrow\; \frac{v_{S1} - v_-}{R_1} + \frac{v_{S2} - v_-}{R_2} + \frac{v_{S3} - v_-}{R_3} = \frac{v_- - v_o}{R_f} + i_-$$

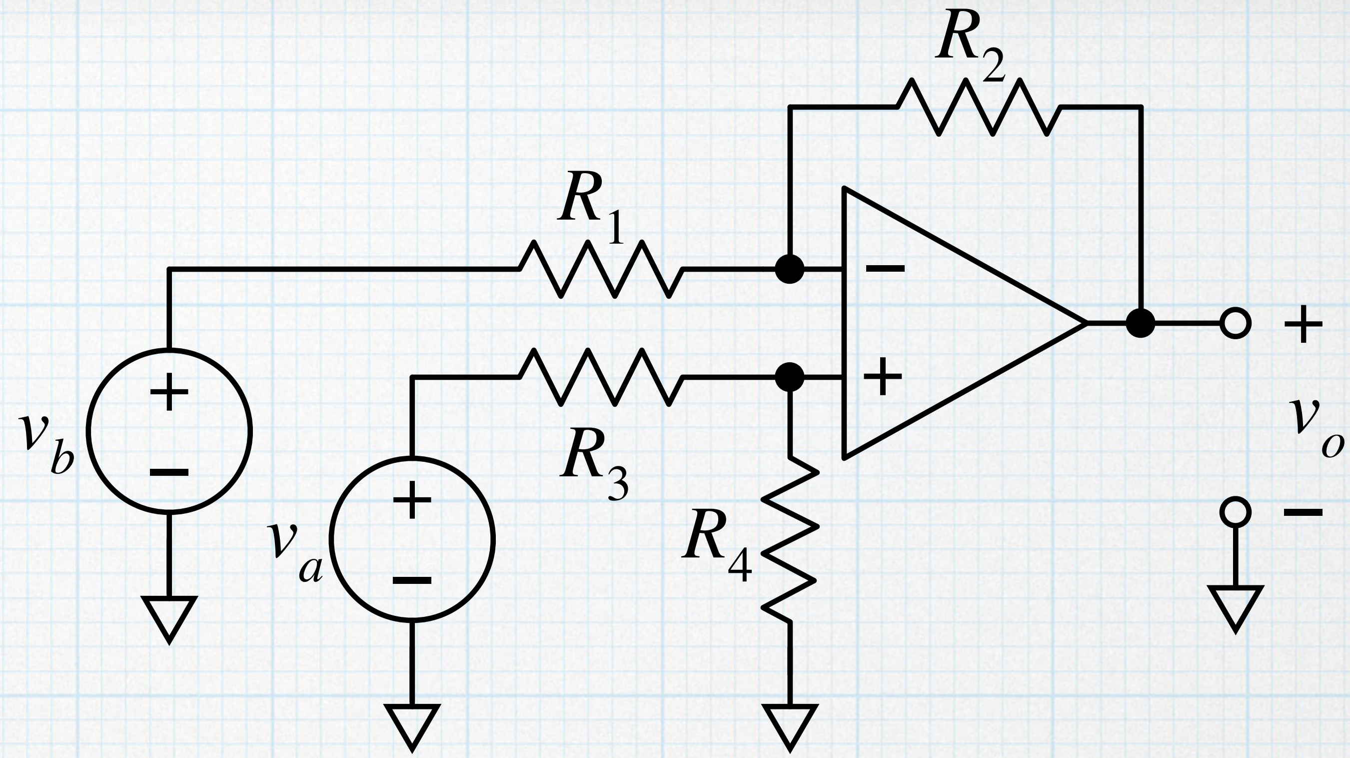
Apply the op amp rules: $i_- = 0$ and $v_- = v_+ = 0$ (virtual ground)

$$\frac{v_{S1}}{R_1} + \frac{v_{S2}}{R_2} + \frac{v_{S3}}{R_3} = \frac{-v_o}{R_f}$$

$$v_o = -\frac{R_f}{R_1}v_{S1} - \frac{R_f}{R_2}v_{S2} - \frac{R_f}{R_3}v_{S3}$$

The circuit scales (weights) the inputs and sums them. Math! The inverting input is a “summing node”. Useful in many applications.

Difference amp



If we can add, perhaps we can also subtract. Find v_o as function of the two inputs.

Write NV equations at v_- and v_+.

$$\frac{v_a - v_+}{R_3} = \frac{v_+}{R_4} + i_+$$

$$\frac{v_b - v_-}{R_1} = \frac{v_- - v_o}{R_2} + i_-$$

Apply the op amp rules: $i_- = i_+ = 0$ and $v_+ = v_-$.

$$v_+ = \left(\frac{R_4}{R_3 + R_4}\right) v_a$$

$$v_o = \left(1 + \frac{R_2}{R_1}\right) v_- - \left(\frac{R_2}{R_1}\right) v_b = \left(1 + \frac{R_2}{R_1}\right)\left(\frac{R_4}{R_3 + R_4}\right) v_a - \left(\frac{R_2}{R_1}\right) v_b$$

Input v_a is scaled, input v_b is scaled, and then the output is the difference between those two quantities.

Difference amp (con't)

$$v_o = \left(1 + \frac{R_2}{R_1}\right)\left(\frac{R_4}{R_3 + R_4}\right) v_a - \left(\frac{R_2}{R_1}\right) v_b$$

This is an interesting result, but it becomes even more interesting if we play around with the resistors a bit. Start by re-arranging:

$$v_o = \left(1 + \frac{R_2}{R_1}\right)\left(\frac{{R_4}/{R_3}}{1 + {R_4}/{R_3}}\right) v_a - \left(\frac{R_2}{R_1}\right) v_b$$

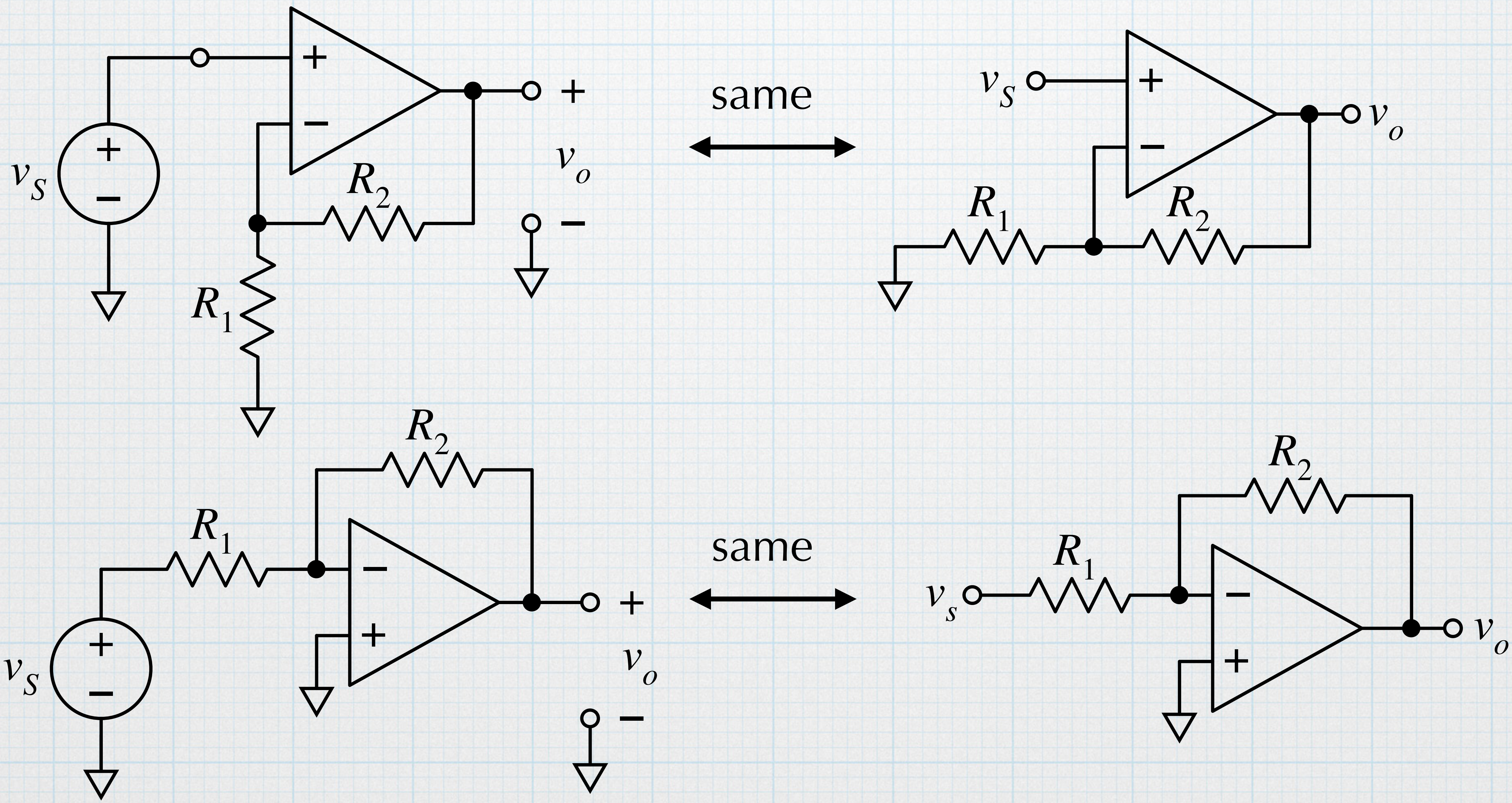
If we choose the resistors such that $R_4/R_3 = R_2/R_1$ then the expression reduces to:

$$v_o = \left(\frac{R_2}{R_1}\right)\left(v_a - v_b\right)$$

The circuit takes the difference between the two inputs and amplifies it. It suppresses *common* voltages and enhances *difference* voltages. For example, suppose $v_a = 10.05$ V = 10 V + 0.05 V and $v_b = 9.95$ V = 10 V – 0.05 V. The two inputs have a common voltage of 10 V and the difference is 0.1 V. If we apply these signals to a difference amp with $R_4/R_3 = R_2/R_1 = 10$, the output will be 1 V. The large common voltage has no effect — only the difference is amplified. Diff amps are useful in sensor applications.

Short-hand notation for voltage sources.

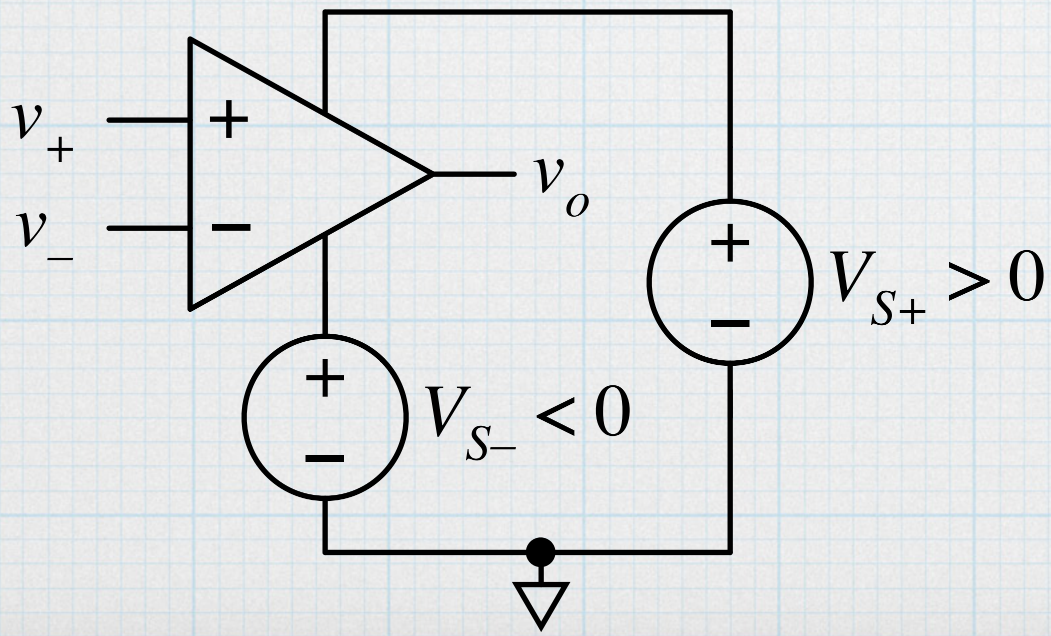
Because electronic circuits always have a pre-defined ground, it really isn’t necessary to draw all the sources. Instead, we use as a short-hand notation and simply label source or supply voltages at a particular node assuming that the indicated voltage is with respect to ground. The simplifies drawing and reading electronic circuit schematics.

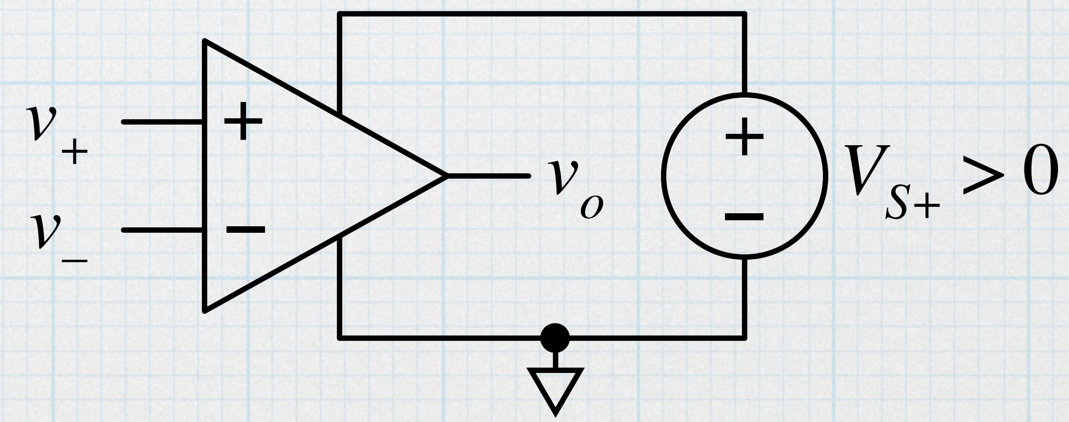


Power supplies

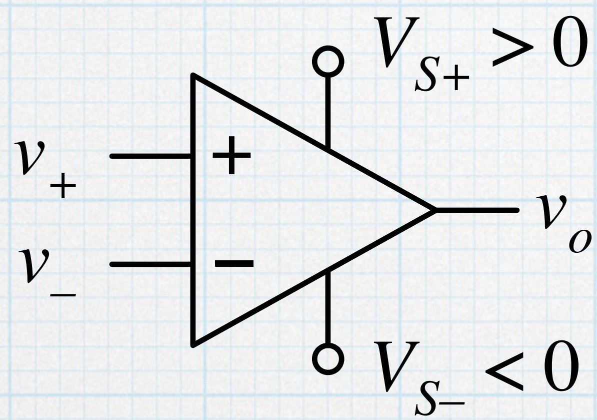
As mentioned previously, an op amp needs DC power supplies in order to function. In a sense, an op-amp is a power conversion device — it takes power from the DC and converts it to extra power added to the signal as it flows from input to output. Without DC supplies, the amp is just a dead element squatting in the middle of the circuit.

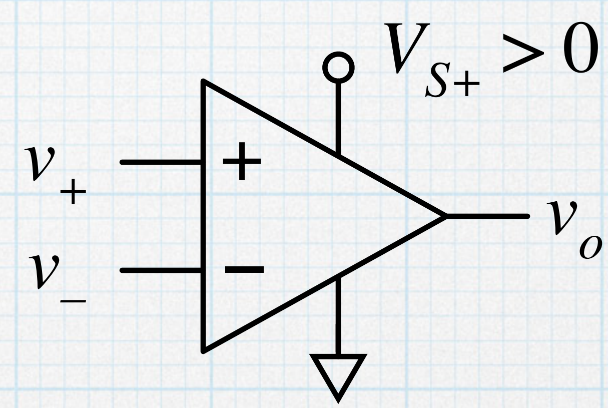
There are two typical supply configurations for op amps: 1) two supplies — one positive and one negative, usually matched — with ground defined as the common point between them, or 2) a single positive supply with ground. The op amp will have pins for connecting to the supplies. Op amp (usually) do not have a pin that connects to ground, but in the single-supply case, the negative supply pin will be connected to ground. The single supply case could be viewed as having $V_{S-} = 0$.





Including power supplies in the schematic become unwieldy, especially in circuits with multiple amps. Usually, the amps will share the same power supplies. If we want to show power supply connections explicitly, we can use the short-hand notation introduced earlier, where we simply label the voltages at the nodes.

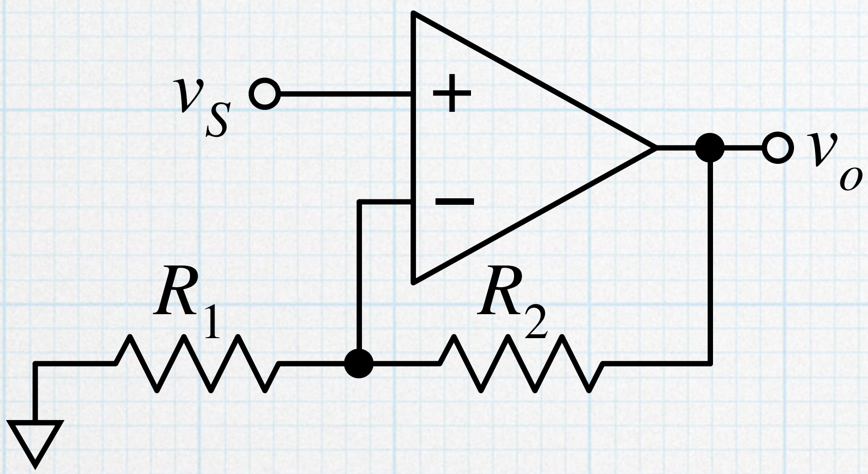




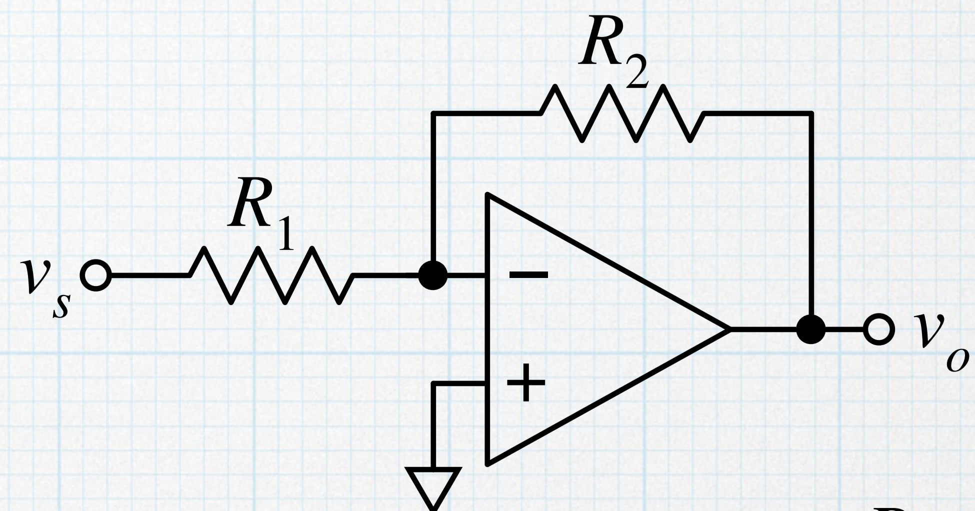
More typically, the power supply connections are not shown at all, as we did in all of the earlier examples. It is assumed that if the op amps are working, there must be power supplies available to make that happen.

Combinations of op amps

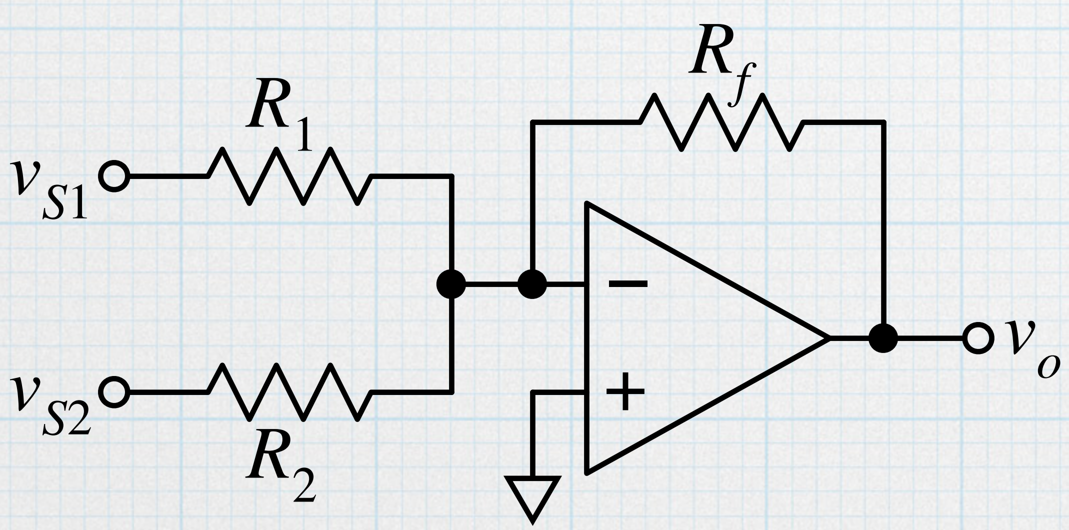
There are four basic op-amp configurations: non-inverting, inverting, summing, difference. (The other examples are variations on these.)



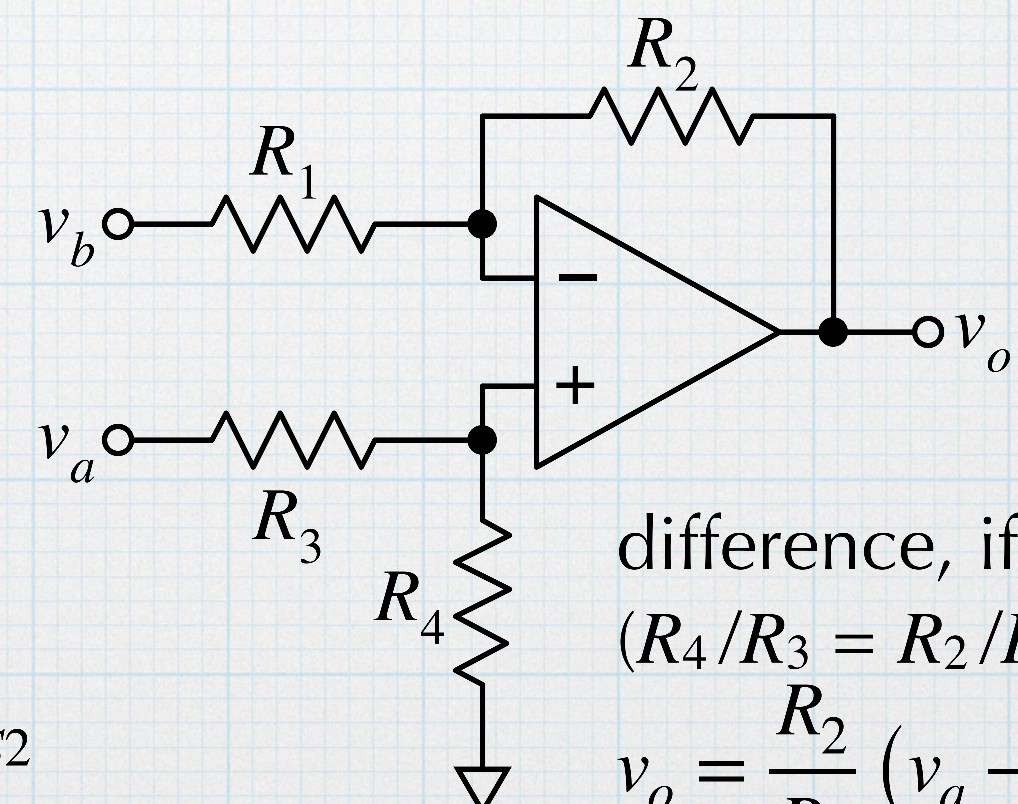
non-inverting, $G = 1 + \frac{R_2}{R_1}$



inverting, $G = -\frac{R_2}{R_1}$



summing, $v_o = -\frac{R_f}{R_1} \cdot v_{S1} - \frac{R_f}{R_2} \cdot v_{S2}$

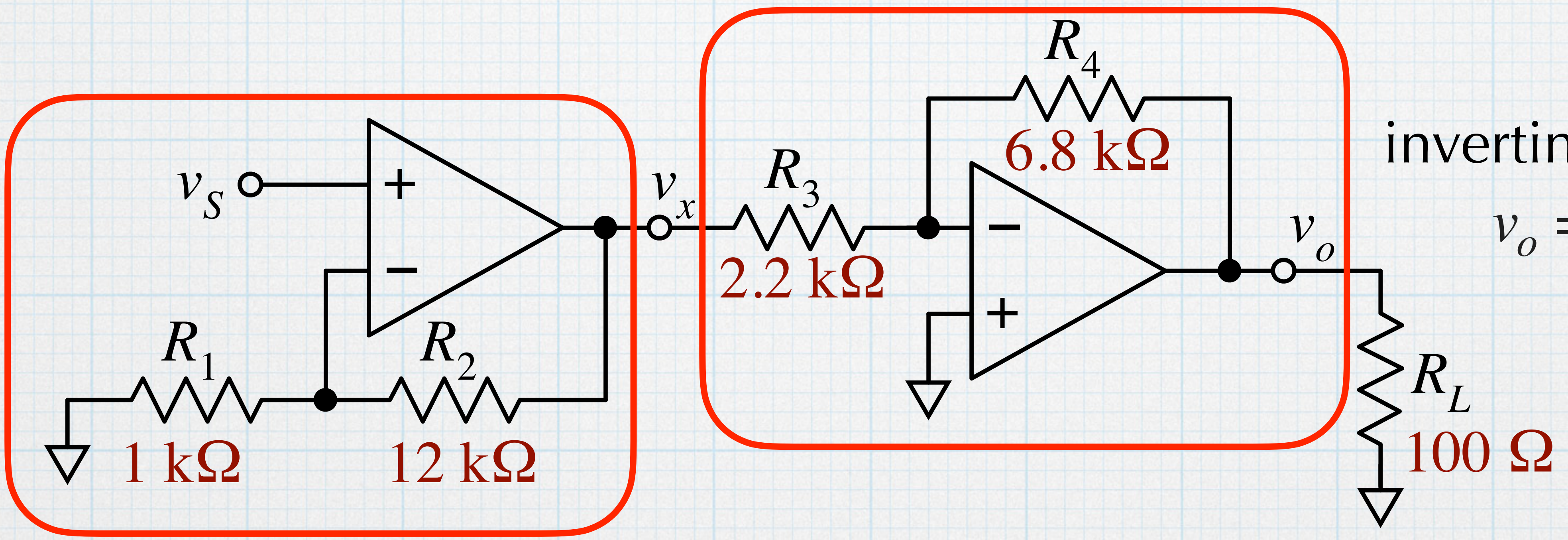


difference, if balanced ($R_4/R_3 = R_2/R_1$), then

$v_o = \frac{R_2}{R_1}\left(v_a - v_b\right)$

We can mix and match the basic configurations to achieve all sorts of circuit functions. The combining of op amp circuits works well because of the third feature of an ideal op amp: zero output resistance. With no output resistance, we can (in principle) connect anything to the output and not have to worry about mismatched input and output resistors affecting the voltage. If $R_o = 0$, the voltage divider ratio is always 1.

Consider the simple cascaded pair of amp circuits shown below.



inverting $v_o = -\dfrac{R_2}{R_1} v_x$

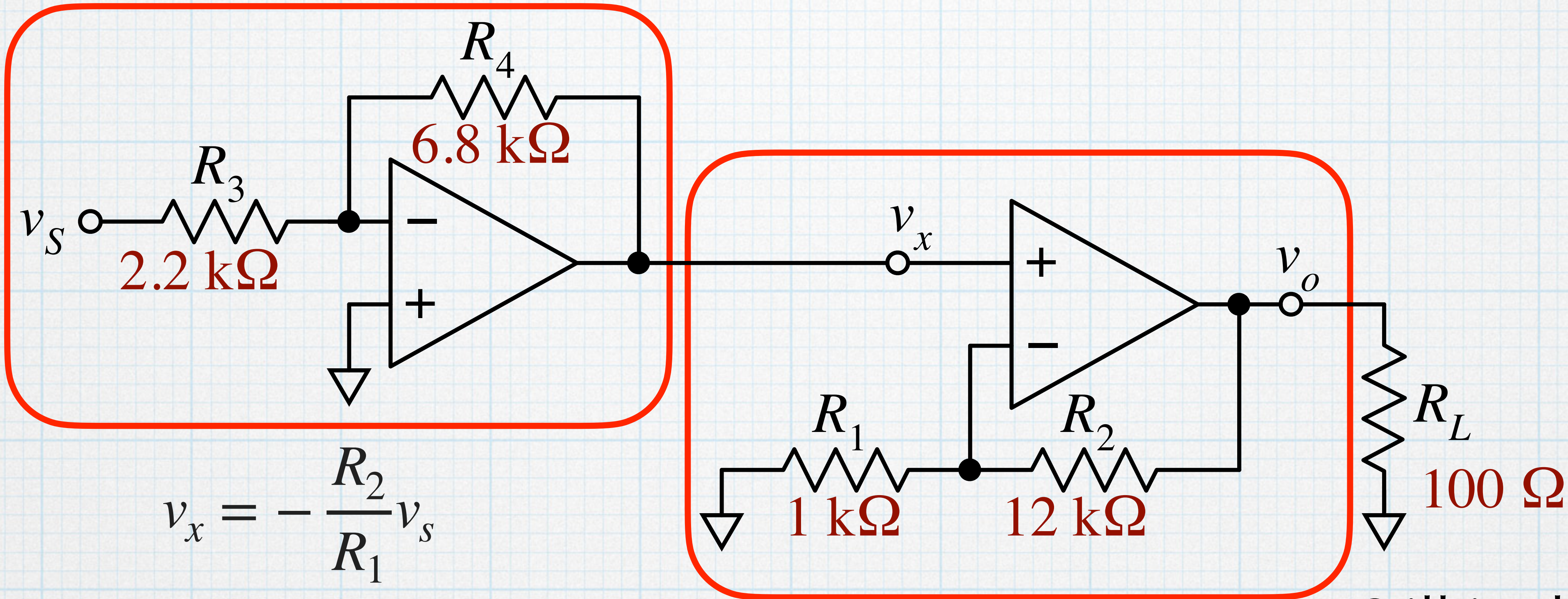
non-inverting $v_x = \left(1 + \dfrac{R_2}{R_1}\right) v_s$

Because of the ideal output resistance, the voltage gains can considered piecemeal.

$$v_o = \left(-\frac{R_4}{R_3}\right)\left(1 + \frac{R_2}{R_1}\right) v_s = -40.2 \cdot v_s$$

The load resistor is irrelevant.

Swapping the order of the two amps doesn't change anything.



$$v_x = -\frac{R_2}{R_1} v_s$$

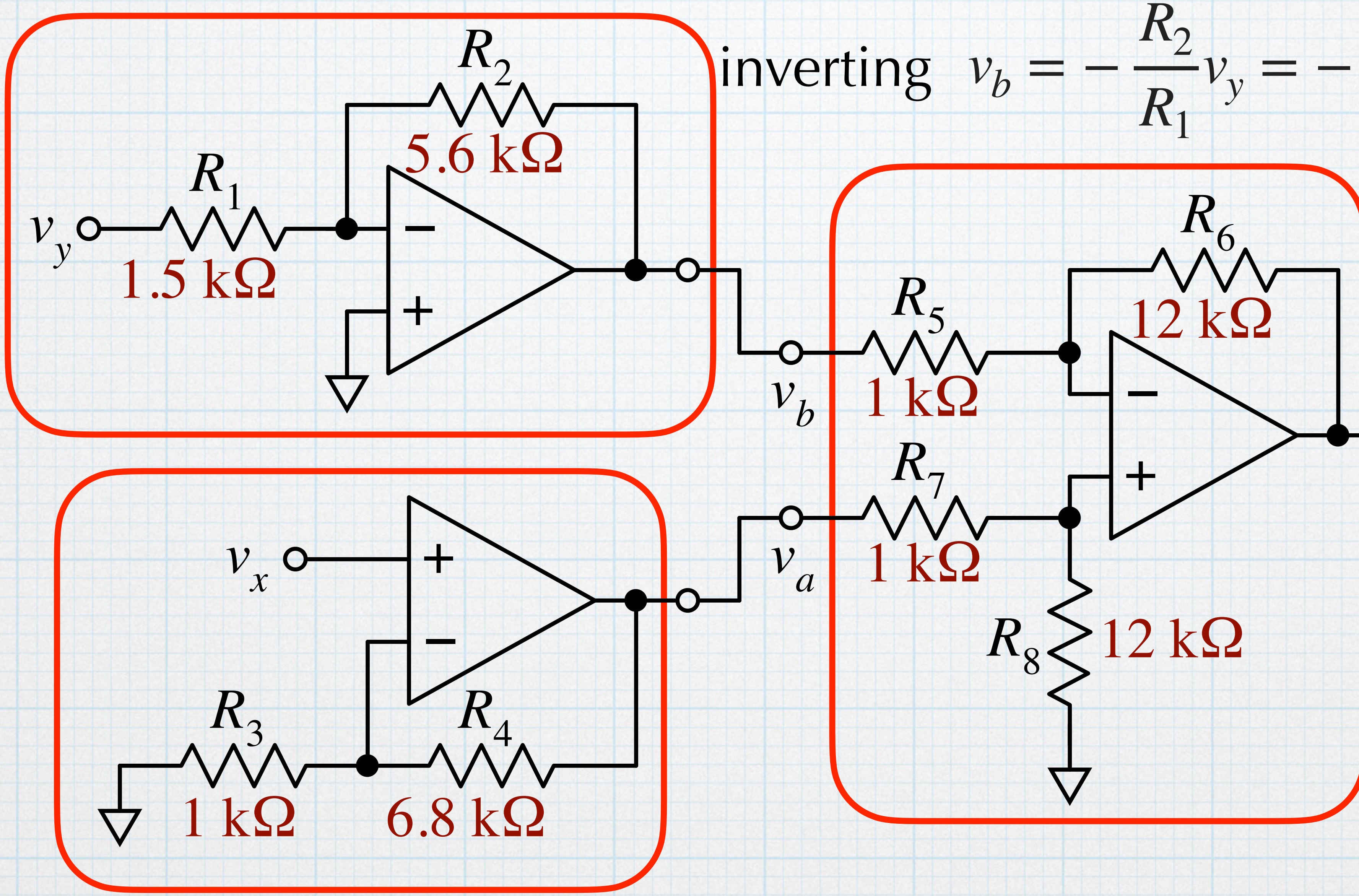
Still irrelevant.

$$v_o = \left(1 + \frac{R_2}{R_1}\right) v_x$$

$$v_o = \left(1 + \frac{R_2}{R_1}\right)\left(-\frac{R_4}{R_3}\right) v_s = -40.2 \cdot v_s$$

Example 5

Find v_o in terms of v_x and v_y. The circuit looks scary, but if we recognize the pieces and take them one at a time, it's not so bad.



inverting $v_b = -\dfrac{R_2}{R_1} v_y = -3.73 \cdot v_y$

balanced difference amp

$$v_o = \frac{R_6}{R_5}\left(v_a - v_b\right)$$
$$= 12\left(v_a - v_b\right)$$

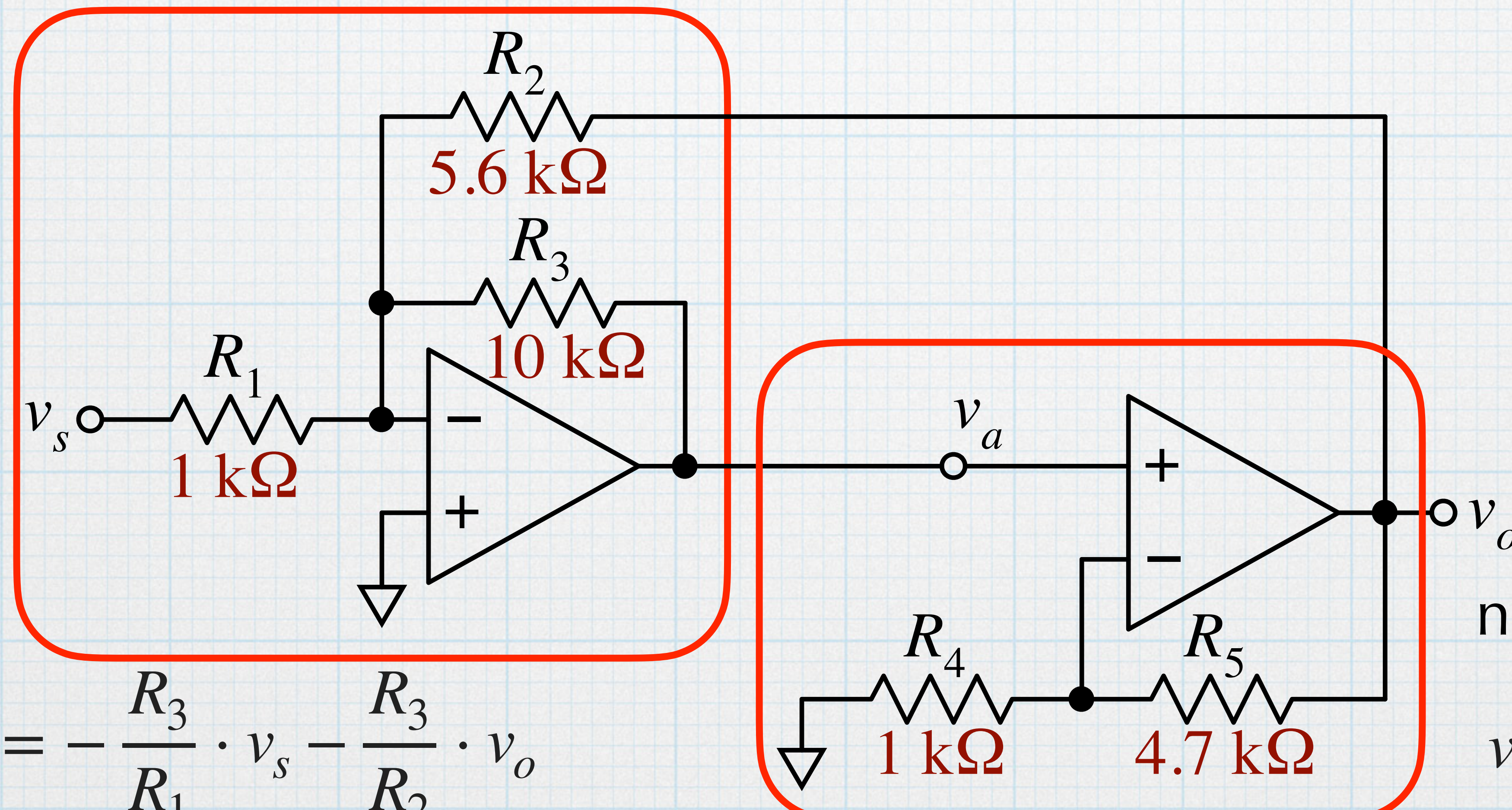
non-inverting $v_a = \left(1 + \dfrac{R_4}{R_3}\right) v_x = 7.8 \cdot v_x$

$$v_o = 12\left[7.8 \cdot v_x - \left(-3.73 \cdot v_y\right)\right] = 93.6 \cdot v_x + 44.8 \cdot v_y$$

Example 6

For the circuit below, calculate the total gain, $G = v_o/v_S$.

At first glance this looks like an inverting amp followed by a non-inverting. But what is going on with R_2? On further consideration, we realize that the left amp is actually a summing amp with feedback resistor R_3, and two input, v_S (through R_1) and v_o (!!) through R_2. Once we see that, the analysis is easy.



summing

$$v_a = -\frac{R_3}{R_1} \cdot v_s - \frac{R_3}{R_2} \cdot v_o$$
$$= -10 \cdot v_s - 1.79 \cdot v_o$$

non-inverting

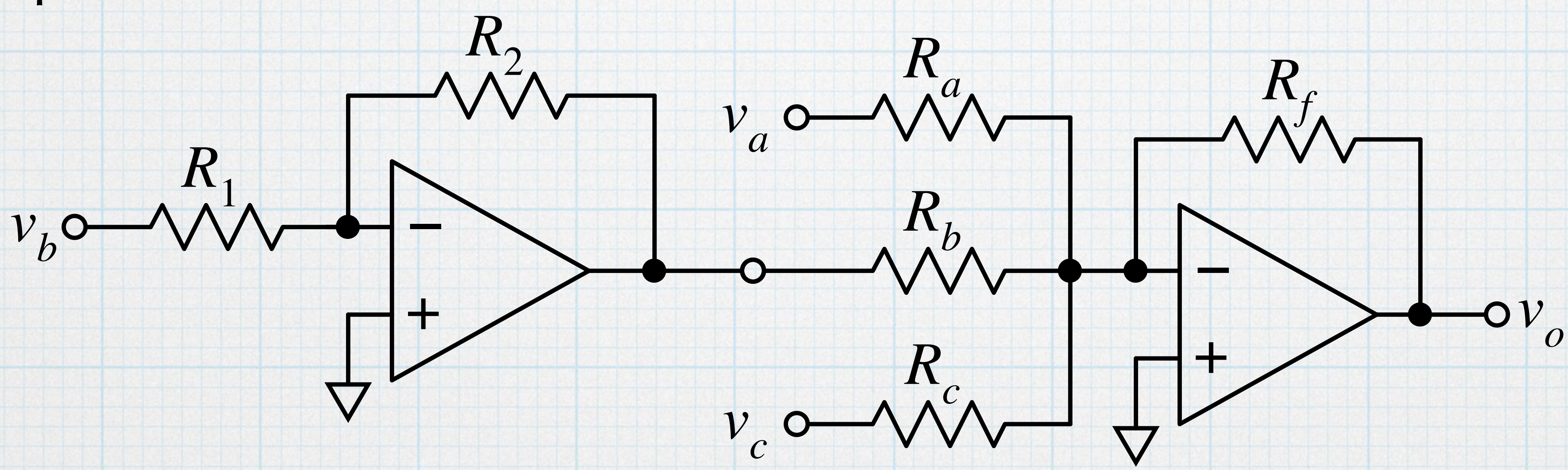
$$v_o = \left(1 + \frac{R_4}{R_3}\right) v_a$$
$$= 5.7 \cdot v_a$$

$$v_o = 5.7\left(-10 \cdot v_s - 1.79 \cdot v_o\right) \rightarrow \frac{v_o}{v_s} = -5.09$$

Example 7

Devise a circuit with ideal op-amps and resistors that will take three inputs, v_a, v_b, v_c and combine them to produce an output voltage according to the function, $v_o = -6.67 \cdot v_a + 12 \cdot v_b - 3 \cdot v_c$.

There are many possible solutions. It looks like a three-input summing amp would come close, except for the positive middle term. A way to handle that would be to invert v_b first before feeding it to the summing amp. The circuit below should work.



$$v_o = -\frac{R_f}{R_a} \cdot v_a + \left(\frac{R_f}{R_b}\right)\left(\frac{R_2}{R_1}\right) v_b - \frac{R_f}{R_c} \cdot v_c$$

After some trial-and-error in choosing resistors, the following set seems to work: $R_f = 10\ \mathrm{k\Omega}$, $R_a = 1.5\ \mathrm{k\Omega}$, $R_c = 3.3\ \mathrm{k\Omega}$, $R_2 = 1.2\ \mathrm{k\Omega}$, $R_1 = 1\ \mathrm{k\Omega}$, and $R_b = 1\ \mathrm{k\Omega}$.