

Linear Regulators

Advantages of linear regulators:

- Simple to use
- Inexpensive
- Electrically “quiet”

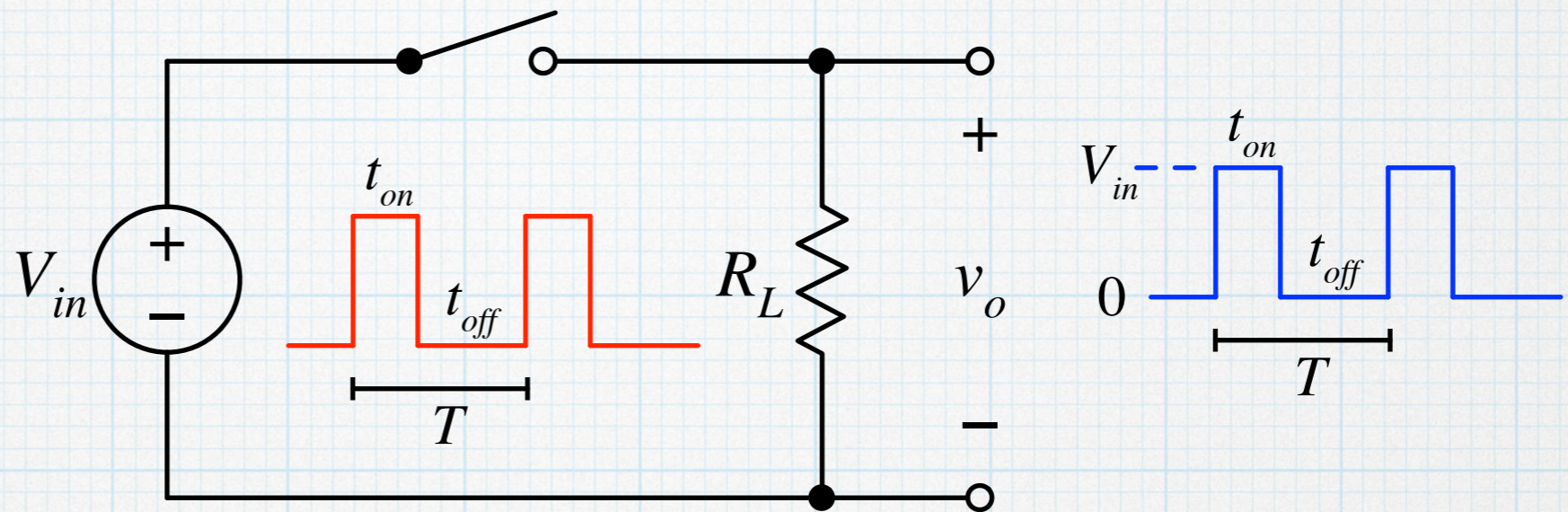
Disadvantages

- Only step-down (buck) operation
- Inefficient when v_o is much smaller than V_{in} and/or high output currents are required. Much of the excess energy is wasted as heat in the output transistor of the regulator.

The crux of the problem is that the output transistor is operating in active mode, with a large voltage drop between source and drain (or collector and emitter). If there is current flowing through the transistor, there will be energy loss. To improve efficiency, the transistor must be used in switched mode. For an ideal switch, there is no power dissipation — when the switch is open there can be a large voltage but there is no current and when the switch is closed there can be large current but there is no voltage. Transistors are not ideal, but they are much more efficient when operating as switches.

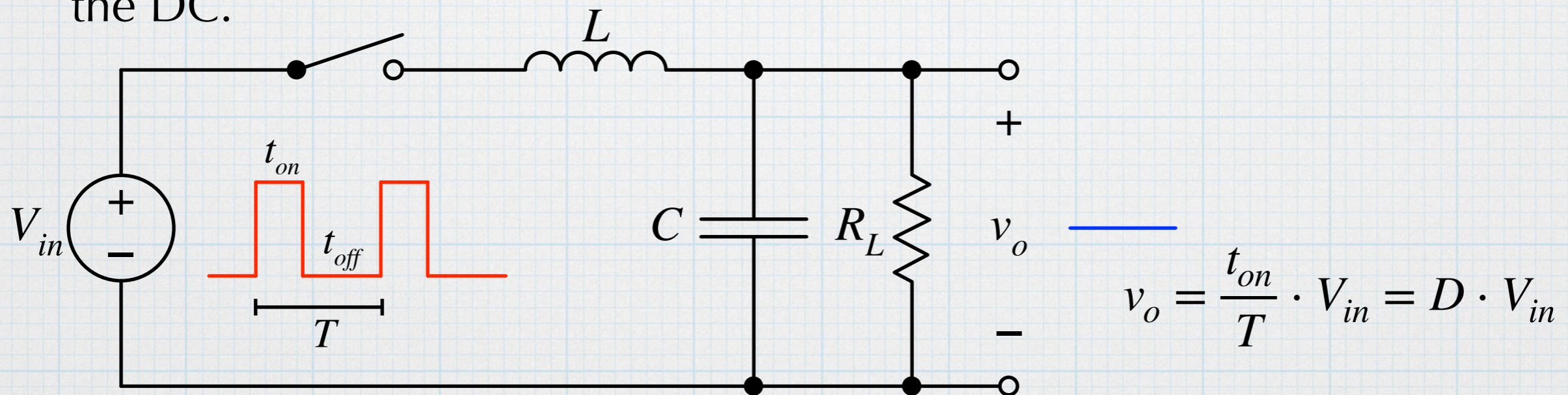
The key to switched-mode operation is PWM.

Take the raw DC input and use a pwm-controlled switch to “chop it up”. The output is a square wave.



$$\text{Duty cycle } D = \frac{t_{on}}{T}$$

Add a low-pass filter it to remove the high-frequency parts, leaving only the DC.



The output is a DC voltage, lower than the input and controlled by pulse-width modulation of the switch.

Switched-mode regulators (SMRs)

The previous slide gives a very high-level view of an SMPS. Taking a DC voltage and breaking it up into pieces seems a little strange and possibly counter-productive. But using the transistor in a switched fashion greatly reduces the power dissipated in it. The advantage in power savings greatly outweighs the disadvantage of a more complicated implementation. The obvious advantages:

- Less power wasted. Lower energy costs. Better battery life. Better for the planet.
- Since the transistor is dissipating less power, it can be smaller, which means it is cheaper and more compact.
- Less heat being generated means less heat sinking required, which also translates to lower cost and lower size and weight.

There is another important and non-obvious advantage to “switchers”:

- In addition to the “step-down” voltage described on the previous slide, it is also possible to “step-up” ($v_o > V_{in}$) and “invert” ($v_o = -V_{in}$) — operations that are impossible with linear regulators. The operation of step-up (or boost) and inverting regulators is not intuitive — we must look at the operation of the circuits somewhat differently than the simplified “switch-and-filter” description of the previous slide.

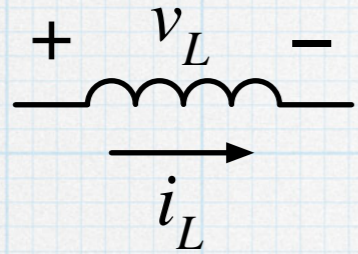
The inductor redeems itself

In past electronics classes, we always denigrated the poor inductor. We tried to ban them from electronics circuits, because inductors are: bulky (in both size and weight), costly (there are lots and lots copper and iron atoms), and lossy (due to series resistance). These all go against our electronics “mantra” — higher speed, smaller size, lower price, and lower power. In order to avoid inductors, we even went to the extreme of making fake inductors using tricky op amp circuits.

In our simplified description of the SMR from two slides back, the inductor was used as part of a filter. We might consider trying to use an RC filter (of whatever order) in place of the LC filter. But low-pass RC filters have resistors in series and capacitors in shunt, meaning that the output current would have to flow through the resistors. An RC circuit may be a good filter, but it would be lossy. In EE 201, we described the inductor as a fancy short circuit, and an ideal inductor would have no loss. So if we can find a good “real” inductor that has minimal series resistance, the LC filter will be better than an RC filter.

However, viewing the inductor as simply a filter component does not give a complete picture of how switched-mode regulators work. The operation is more complicated and subtle, and the inductor is central to the story. We need to view the inductor as an energy storage element and then recall that there are complications when we try to change inductor current rapidly.

Brief review of inductor properties.



$$v_L = L \frac{di_L}{dt}$$

$$i_L(t) = \frac{1}{L} \int_0^t v(t') dt' + i_L(0)$$

An inductor is basically a coil of wire, probably wrapped around a core of magnetic material. It is characterized by the inductance, L . The unit of inductance is henries (H) ($= \text{V}\cdot\text{s}/\text{A}$). The inductance depends on: a) the number of turns in the coil, b) diameter of the coil, c) the diameter of the wire, the separation of the wire between each turn of the coil, d) the permeability of the core material, and e) other things. (It's a bit complicated — see EE 311.)

The current through an inductor cannot change instantaneously. Doing so would induce an infinitely large voltage and require an infinite amount of power. Thus, when trying to use an inductor in a switched application, it will be necessary to provide a means for continuous current flow.

An ideal inductor has no resistance, but real inductors, with all of that wire, will have a series resistance, which can be small — a fraction of an ohm — or big (10s of ohms for crappy inductors.) It all depends on the diameter of the wire. Good quality inductors must be bigger! And cost more!

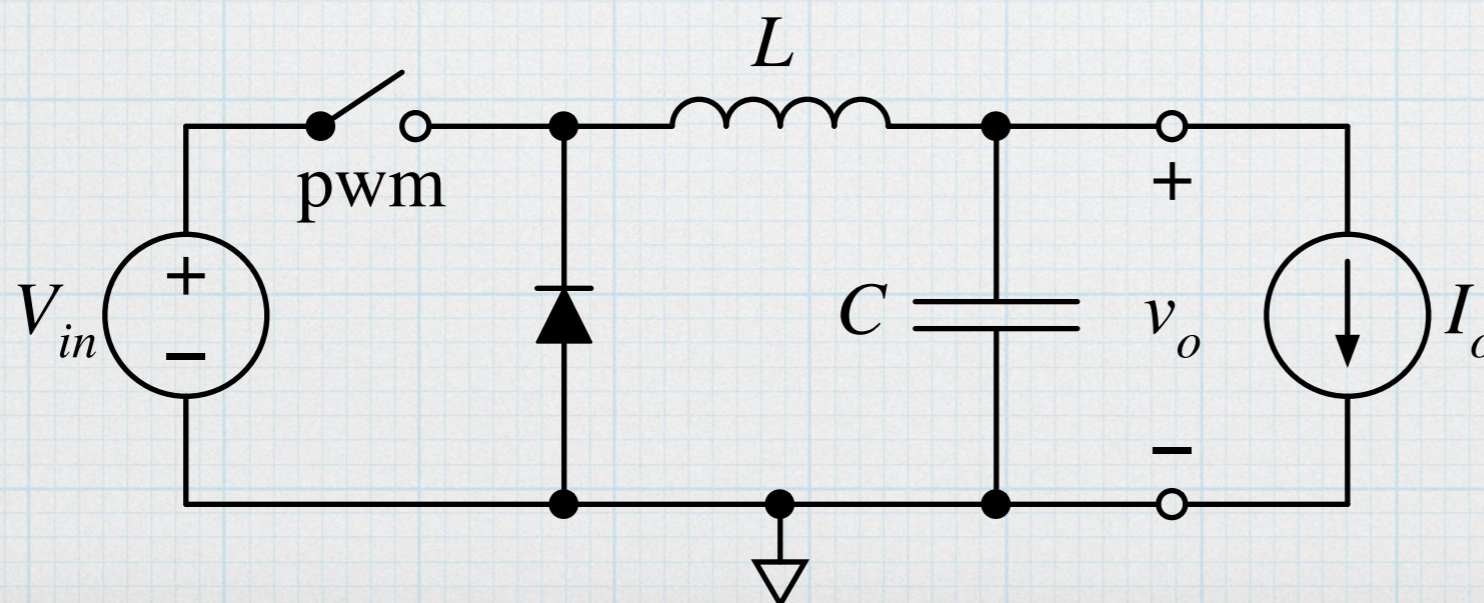
Inductors store energy in the magnetic field created at the center of the coil when current is flowing. The energy is $E_L = \frac{1}{2} Li_L^2$.

Step-down (buck) configuration

This is the most common configuration and matches the behavior of linear regulators. The output voltage is less than the input, $v_o < V_{in}$.

As a first step, we won't solve a bunch of differential equations. Instead we describe the non-linear steady-state operation of the circuit. Here are our assumptions:

- Ideal components (the switch has $R_{switch} = 0$, the inductor has no series resistance, the diode is ideal with a turn-on voltage of $v_D = 0$, and there is no resistance associated with the capacitor).
- The switch is opened and closed by a PWM signal with period T and duty cycle $D (= t_{on} / T)$.
- The input is a constant DC with value V_{in} . The output is also constant DC with value v_o .



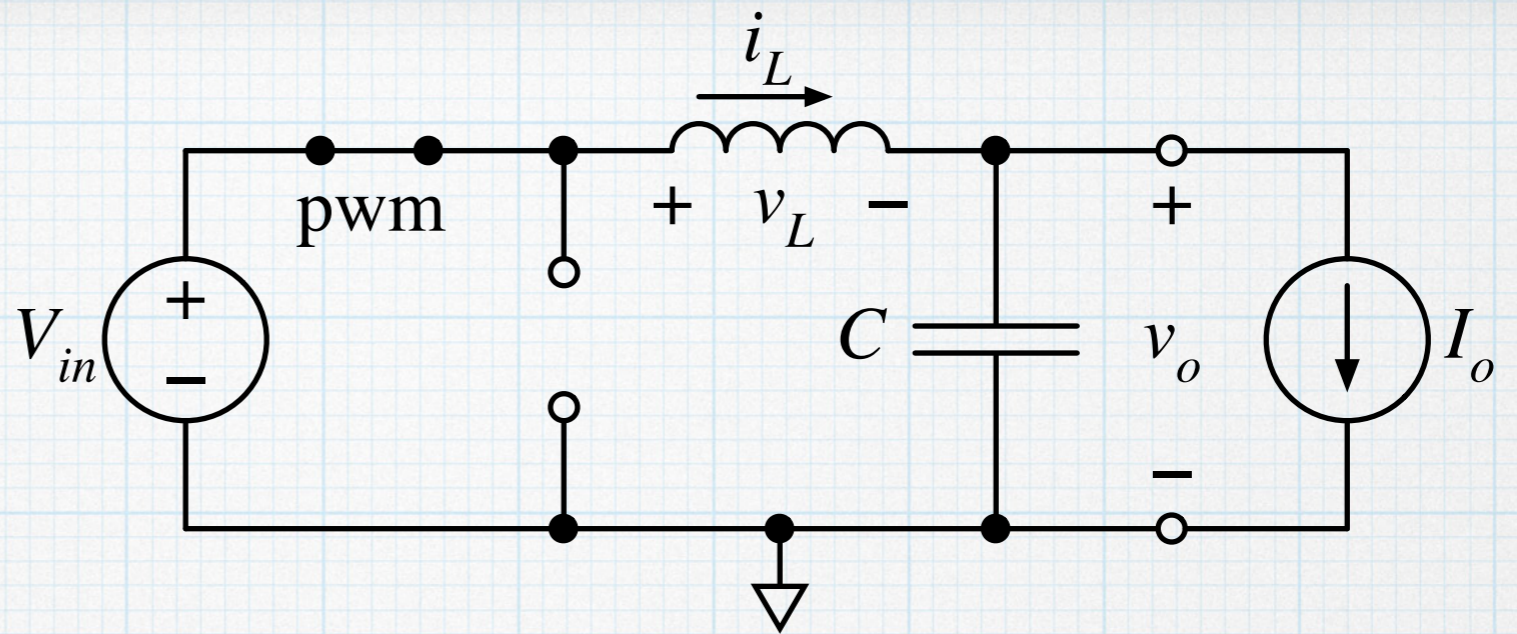
I_o represents the current supplied to the load.

Buck - switch closed

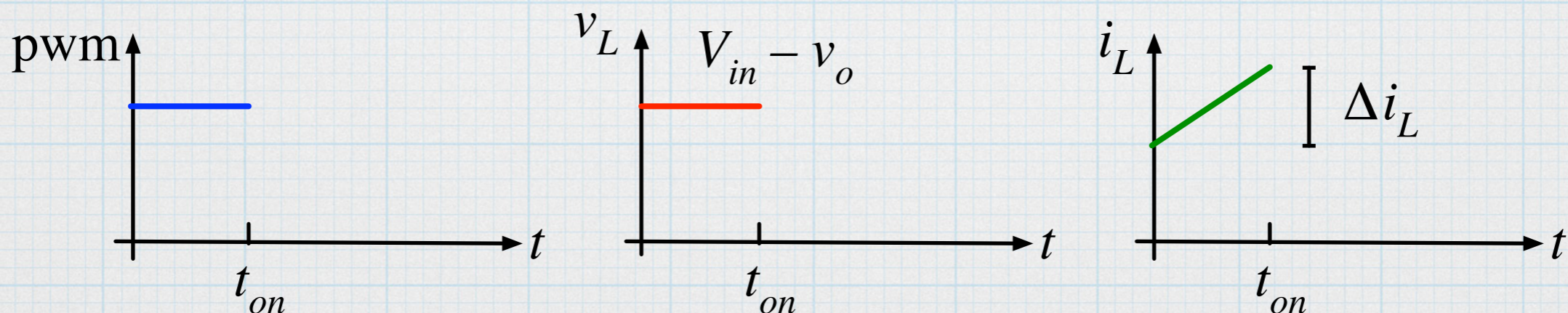
$(0 < t < t_{on})$

Assume an initial inductor current, $i_L(0)$.

With the switch closed:



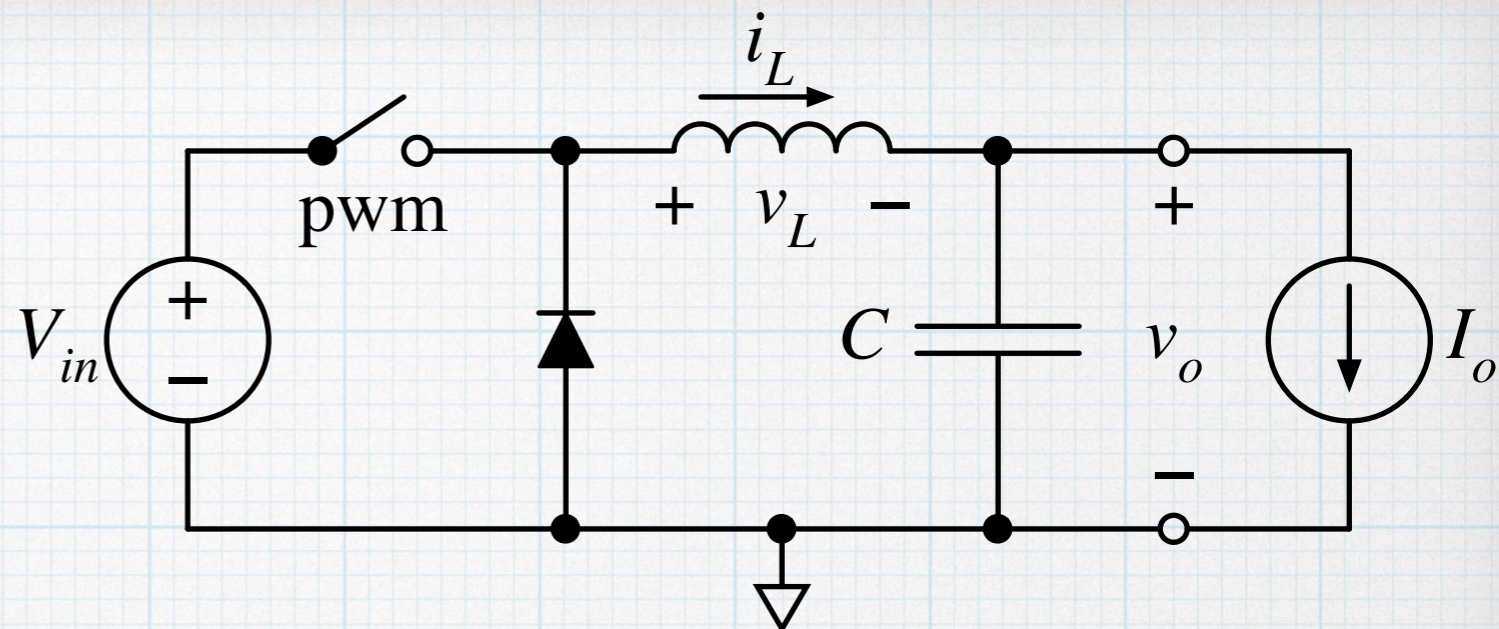
- The left end of the inductor is at V_{in} . The right end will be at v_o . The voltage across the inductor is constant, $v_L = V_{in} - v_o$.
- The diode is reverse-biased, $v_D = -V_{in}$. (So we can ignore it.)
- The inductor current increases with time: $i_L(t) = \frac{1}{L} (V_{in} - v_o) t + i_L(0)$
- The change in the current during the “on” time is: $\Delta i'_L = \frac{V_{in} - v_o}{L} t_{on}$
- The inductor energy increases with time.



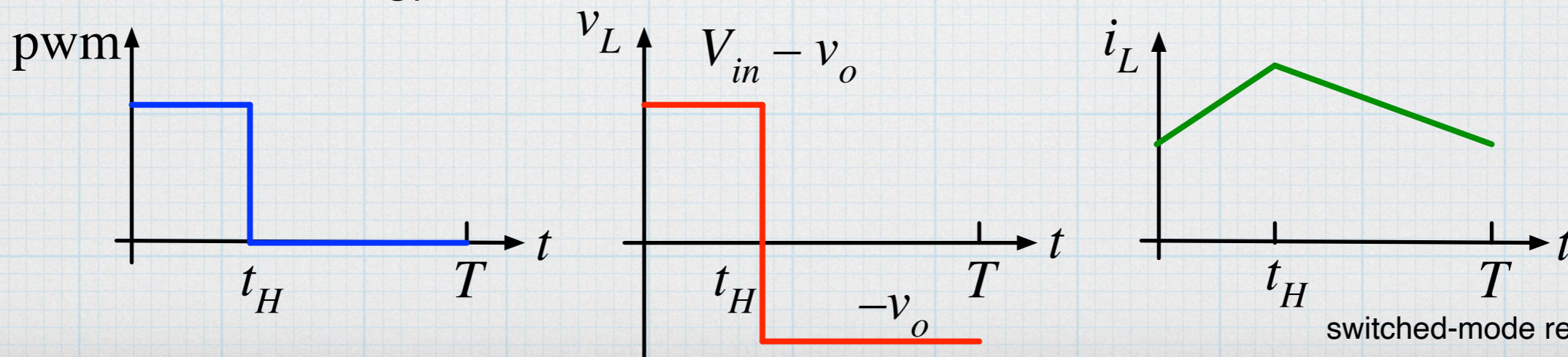
Buck - switch open

$$(t_{on} < t < T)$$

At the end of the previous cycle, the inductor current had ramped to $i_L(t_{on})$. When the switch opens:



- There is no longer a path for current from the source to the inductor. In order to keep the inductor current continuous, the inductor voltage must go sufficiently negative to turn on the diode, creating a path for current. The left end of the inductor must drop to 0 V in order to turn on the ideal diode. Then the voltage across the inductor is negative, $v_L = -v_o$.
- The inductor current decreases with time: $i_L(t) = -\frac{v_o}{L}(t - t_{on}) + i_L(t_{on})$
- The change in the current during the "off" time is: $\Delta i_L'' = -\frac{v_o}{L}(T - t_{on})$
- The inductor energy decreases with time.



That completes one full cycle of the PWM control signal.

If the system is in steady-state, then the power flowing through it must balance. The energy added to the inductor (coming from source) during the “on” phase must balance the energy removed from the inductor (going to the capacitor & load) during the “off” phase.

Since inductor energy depends on current, the increase in current during the first phase must match the decrease during the second phase, $\Delta i'_L = -\Delta i''_L$.

$$\frac{V_{in} - v_o}{L} t_{on} = \frac{v_o}{L} (T - t_{on})$$

$$v_o = \frac{t_{on}}{T} \cdot V_{in} = DV_{in}$$

Since $D \leq 1$ always, the output will be less than the input, as expected.

The source dumps energy into the inductor when the switch is closed, and the inductor, in turn, dumps the energy to the capacitor/load when the switch is opened. The capacitor smoothes the voltage variations due to the see-sawing inductor current.

Buck converter

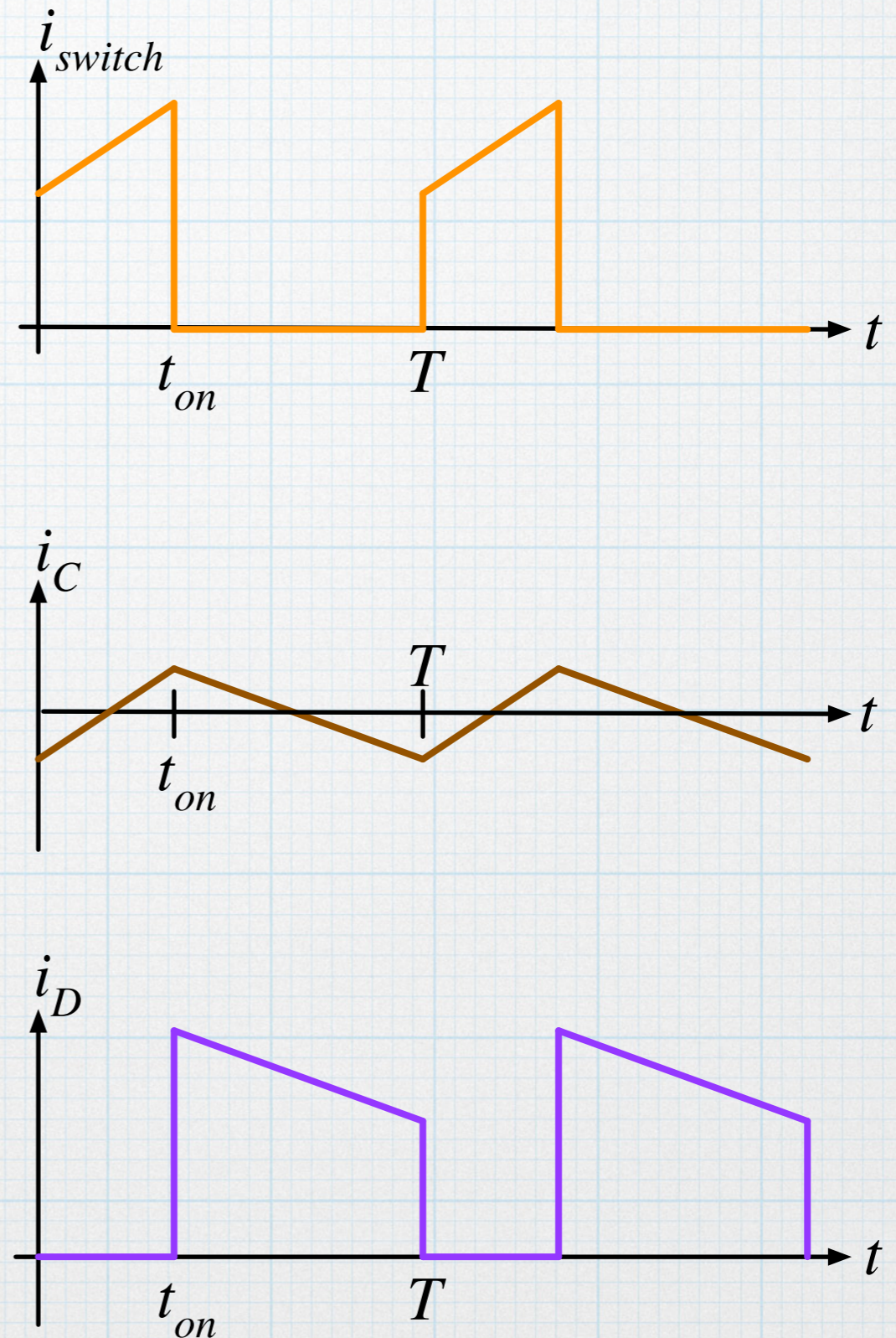
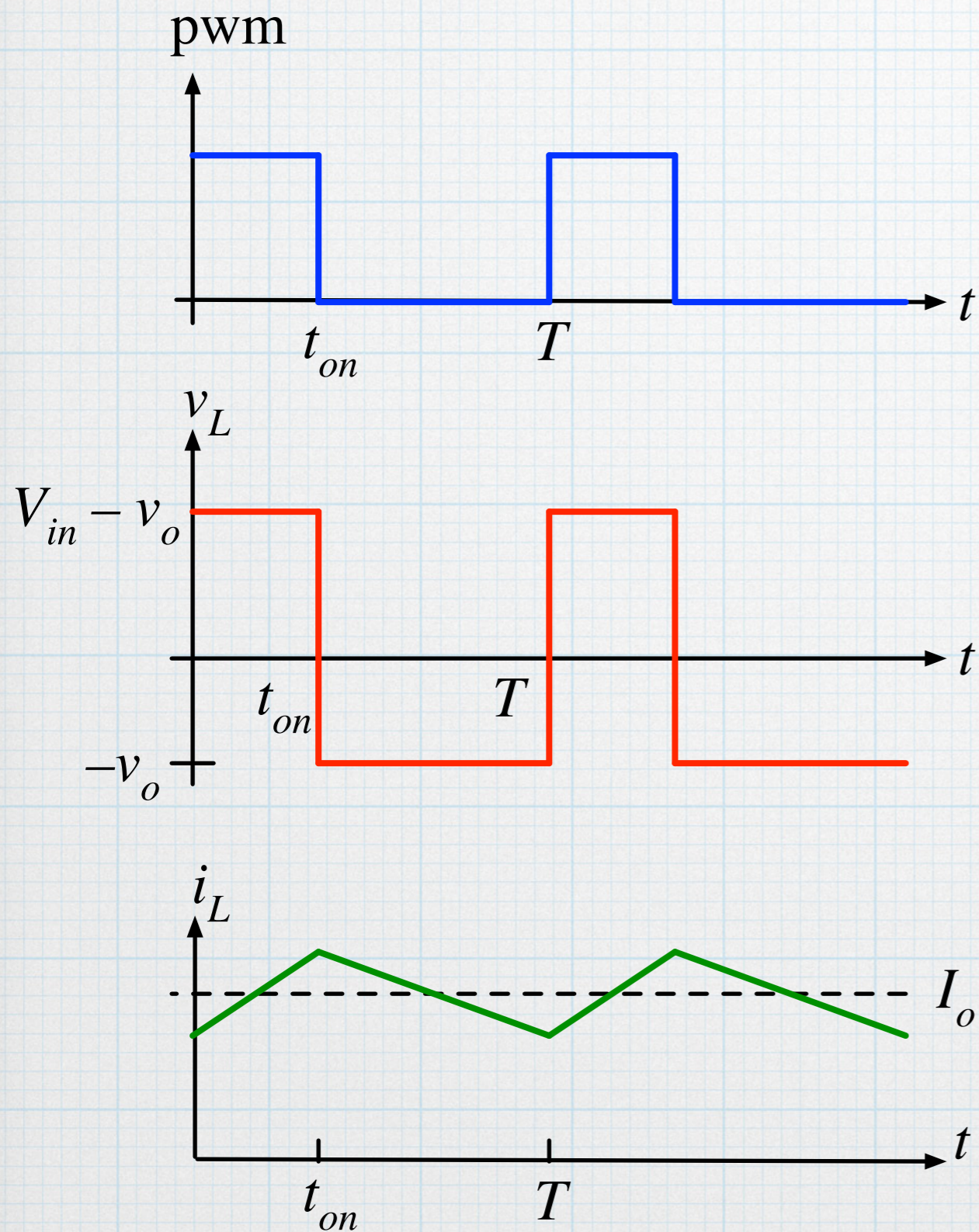
The operation of the converter seems surprisingly simple. It appears that the exact values of the load current, the inductor, and the capacitor value are irrelevant. And in this particular mode of operation with ideal components, that's true. We will see later that the things are a bit more complicated.

Other things to note. (Waveforms on the next slide help visualize.)

1. The current from the source is same as the current in the switch. So the current from the source is pulsing on and off — it is not continuous. When the switch is on, the source current is the same as the inductor current, and it is zero when the switch is off. The average value of the source current is $\langle i_{Vin} \rangle = (t_{on}/T) \cdot I_o = D \cdot I_o$.
2. The average input power is: $P_{in} = V_{in} \cdot i_{Vin} = D \cdot V_{in} \cdot I_o = V_o \cdot I_o = P_{out}$.
3. In other words, under ideal conditions, the converter has perfect efficiency. (That shouldn't be surprising if all the components are ideally lossless.)
4. The diode current is equal to the inductor current when the switch is off. $i_D = i_L$ (off). The average diode current is $\langle i_D \rangle = (t_{off}/T) \cdot I_o = (1 - D) \cdot I_o$.
5. The capacitor current is $i_C = i_L - I_o$. The capacitor current ramps positive and negative corresponding to the ramping of the inductor current.

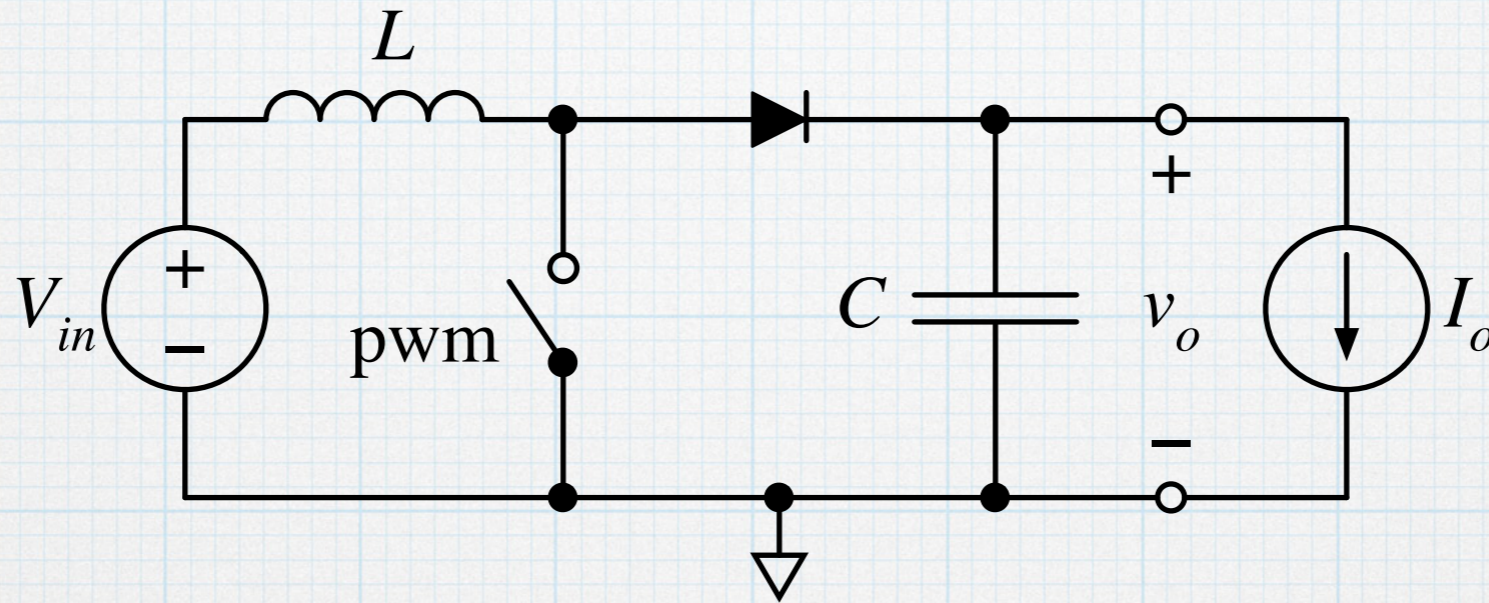
Buck waveforms

In the buck case, the current from the source is the same as the switch current.



Step-up (boost) converter

Now we look at voltage regulation that can be done only by using a switching circuit. By re-arranging the components, we can make a step-up — or boost — converter. The output voltage is bigger than the input.

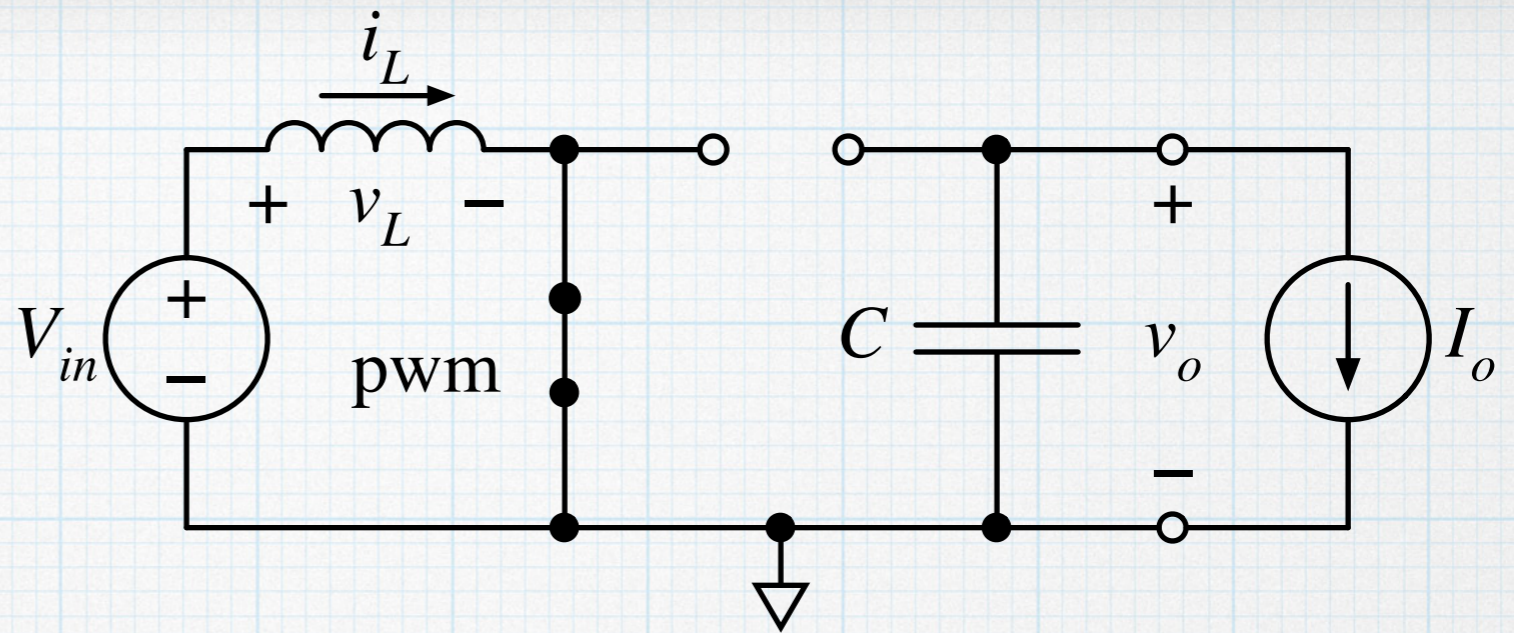


To get an initial view of the how a boost converter functions, we use the same assumptions — ideal components, PWM control of the switch, and constant values for V_{in} and v_o (with $v_o > V_{in}$) — and the same steady-state approach as the buck converter. Look at the switch-closed and switch-opened situations.

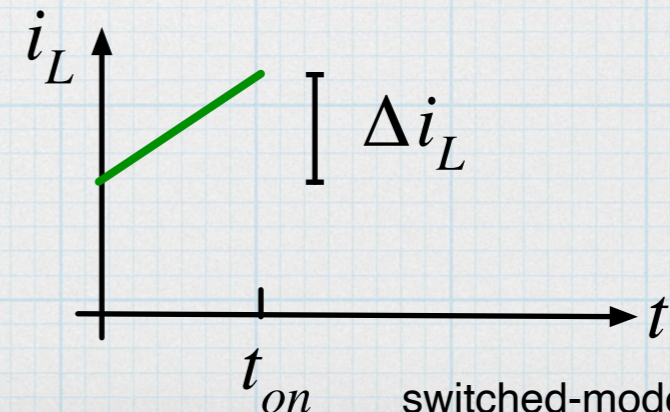
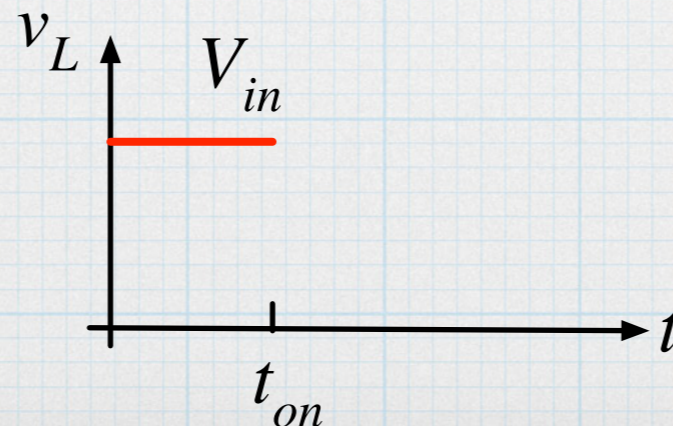
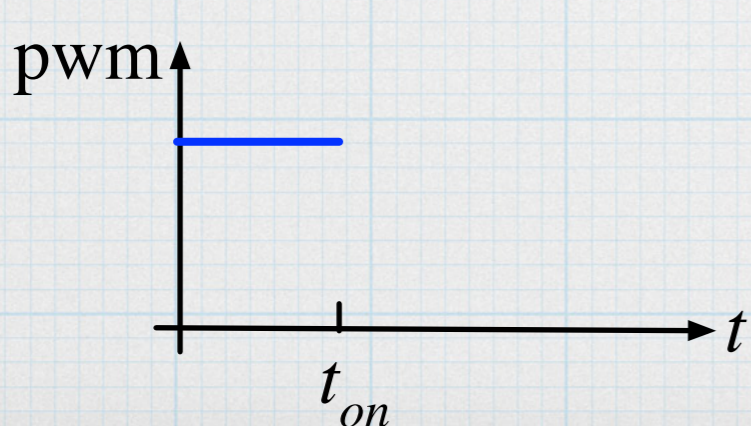
Boost – switch closed

$(0 < t < t_{on})$

Assume that the inductor current starts at some value, $i_L(0)$.



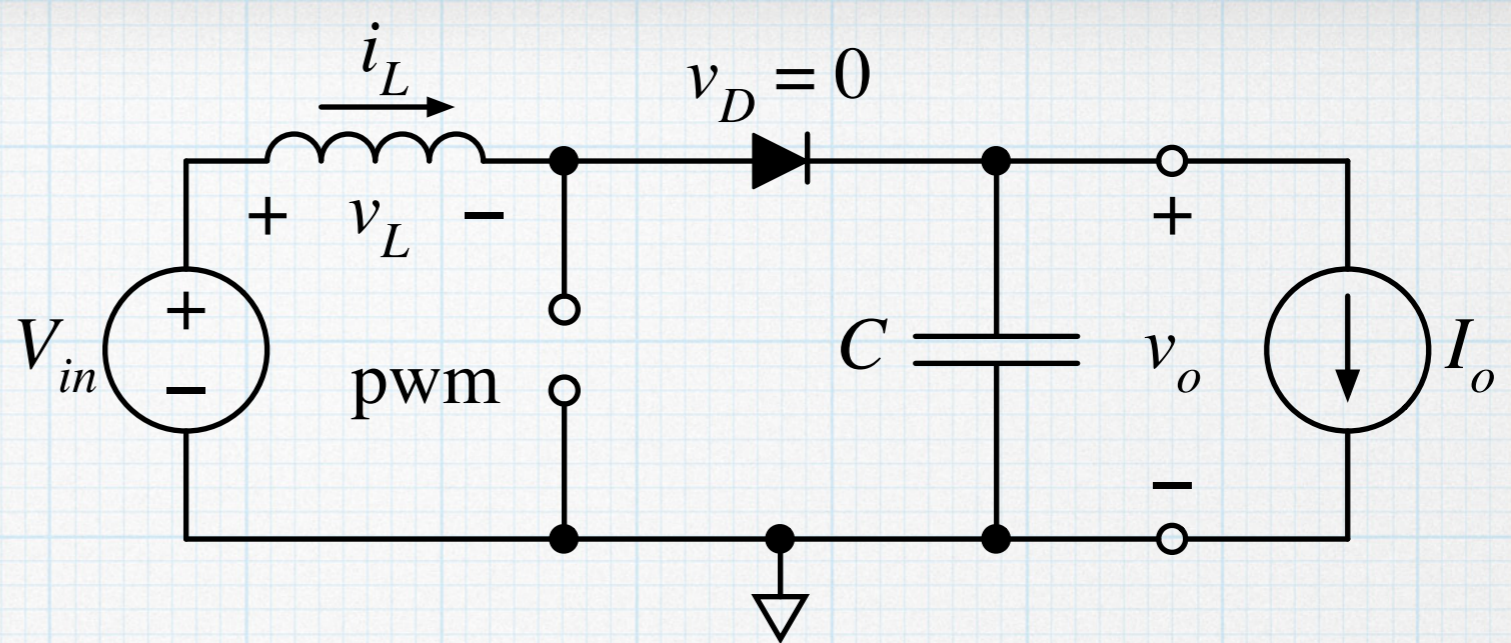
- With the switch shorting to ground, the inductor voltage inductor is $v_L = V_{in}$.
- The diode is reverse-biased ($v_D = -v_o$), effectively disconnecting the input from the output.
- The inductor current increases with time: $i_L(t) = \frac{1}{L}V_{in} \cdot t + i_L(0)$
- The change in the current during the “on” time: $\Delta i'_L = \frac{V_{in}}{L} \cdot t_{on}$
- The inductor energy increases with time.



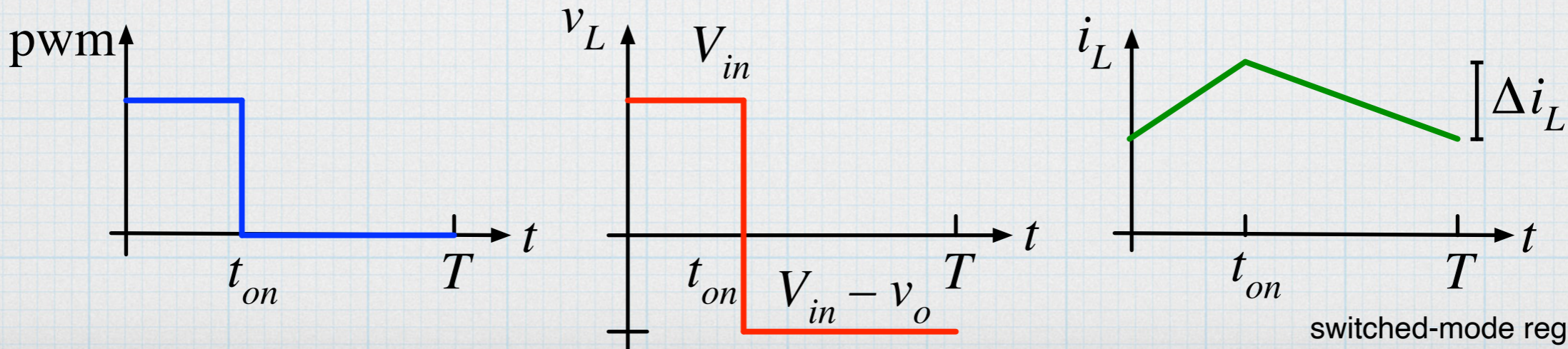
Boost - switch open

$(t_{on} < t < T)$

The inductor current starts at the value $i_L(t_{on})$. When the switch is open:



- The inductor current can't flow through the switch – it must flow through the diode. To turn on the diode, the voltage at the right end of the inductor must increase to v_o . The inductor voltage becomes $v_L = V_{in} - v_o$. Since $v_o > V_{in}$, the inductor voltage is now negative
- The inductor current decreases with time: $i_L(t) = \frac{V_{in} - v_o}{L} (t - t_{on}) + i_L(t_{on})$
- The change in the current during the “off” time is: $\Delta i_L'' = \frac{V_{in} - v_o}{L} (T - t_{on})$
- The inductor energy decreases with time.



As with the buck converter, if the boost converter is in steady-state operation, the power flow must balance. Energy is added to the inductor when the switch is closed, and that energy is transferred to the capacitor and load when the switch is open.

The magnitude of the current increase in the inductor must match the magnitude of the current decrease, $\Delta i'_L = -\Delta i''_L$.

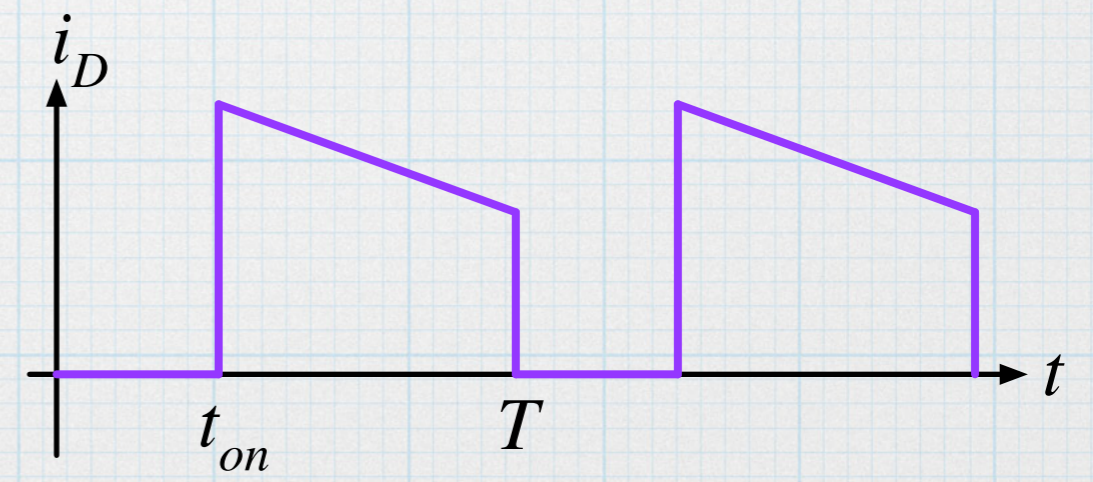
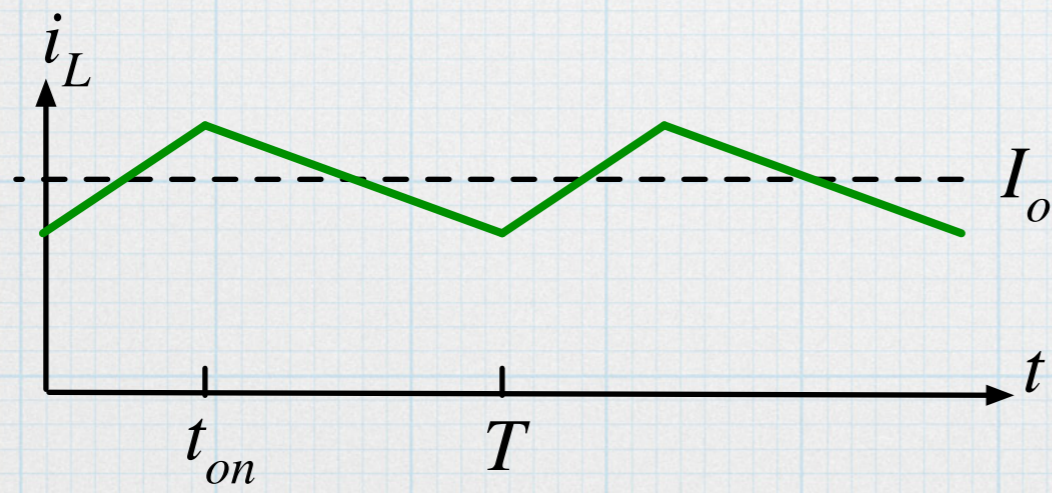
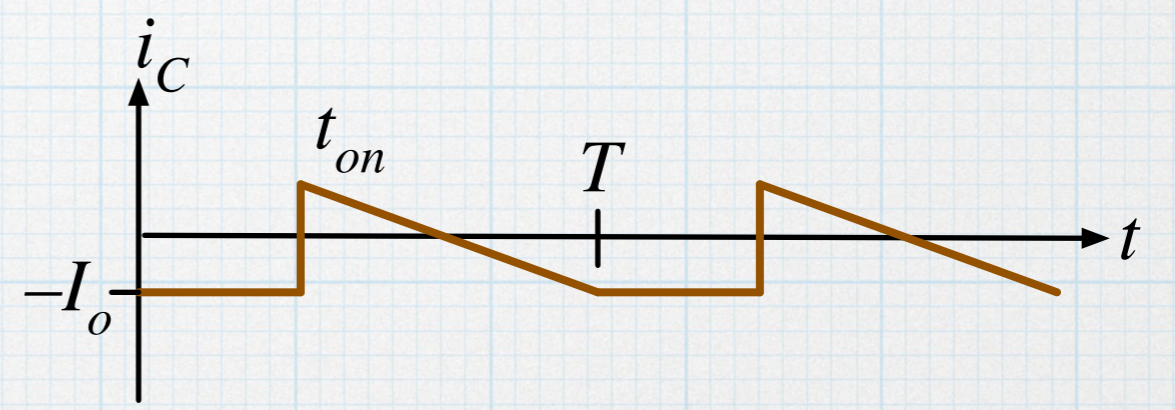
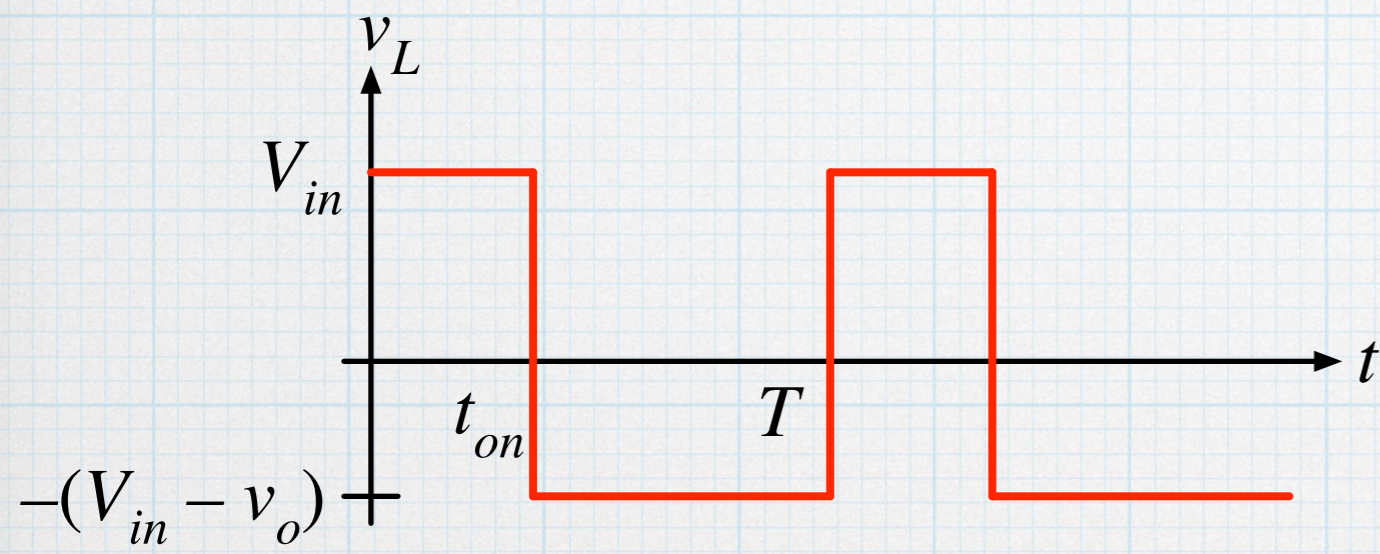
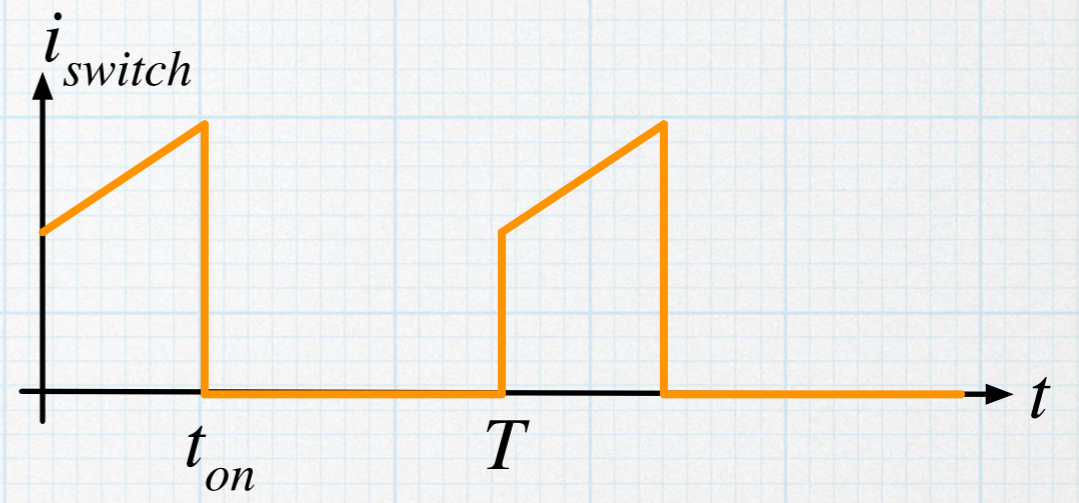
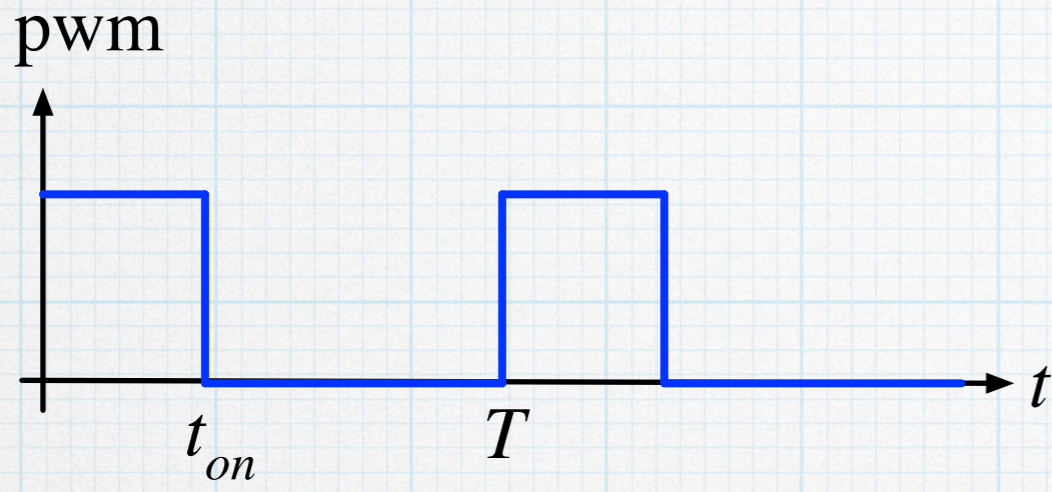
$$\frac{V_{in}}{L} t_H = \frac{v_o - V_{in}}{L} (T - t_{on})$$

$$v_o = \frac{T}{T - t_{on}} V_{in} = \frac{V_{in}}{1 - D}$$

Since the duty cycle is always less than 1, we see that $v_o > V_{in}$, consistent with our starting assumption.

Note that during the on time, when the switch is closed, current flows from the source into the inductor, but with the diode off, there is no current flowing into the cap/load combination. During the on time, the capacitor must supply the entire load current, $i_C = -I_o$. When the switch is off, current flows from the source into the inductor and on to the cap/load. In the boost case, the source current is continuous, but current to the load/cap is not. This is the exact opposite of the buck case.

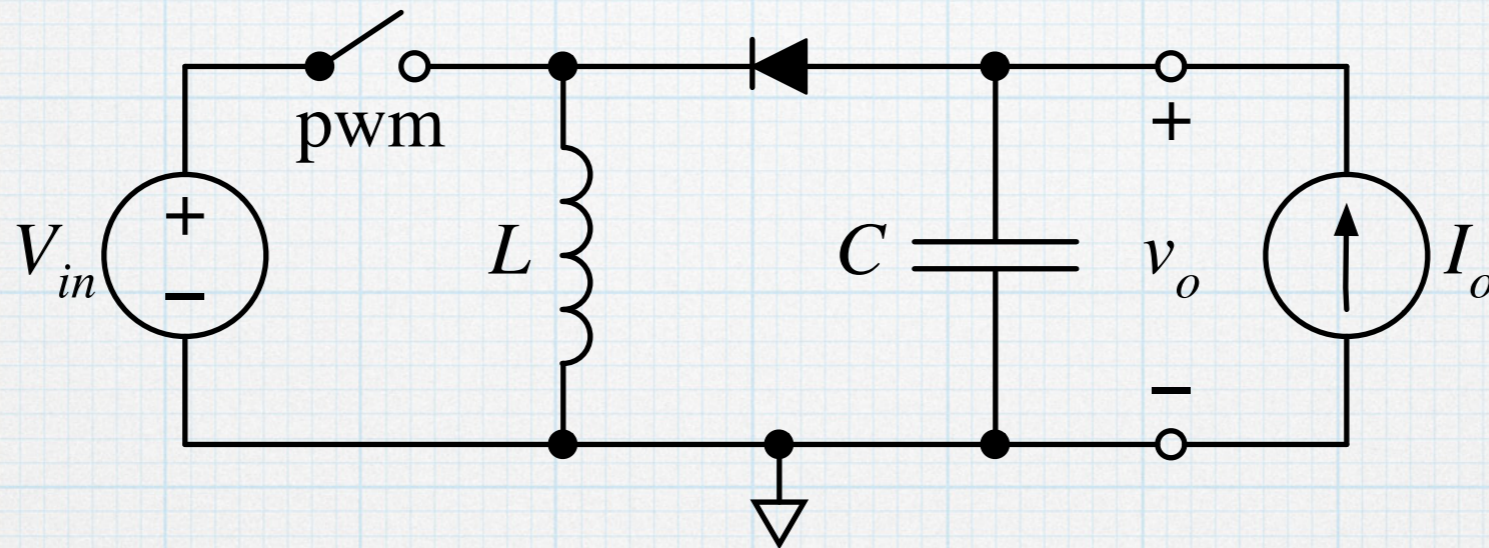
Boost waveforms



In the boost case, the current from the source is the same as the inductor current.

Buck–boost (inverting) converter

With another re-arrangement of the components, we arrive at a switching circuit that takes a positive input voltage and produces a negative output, $v_o < 0$!



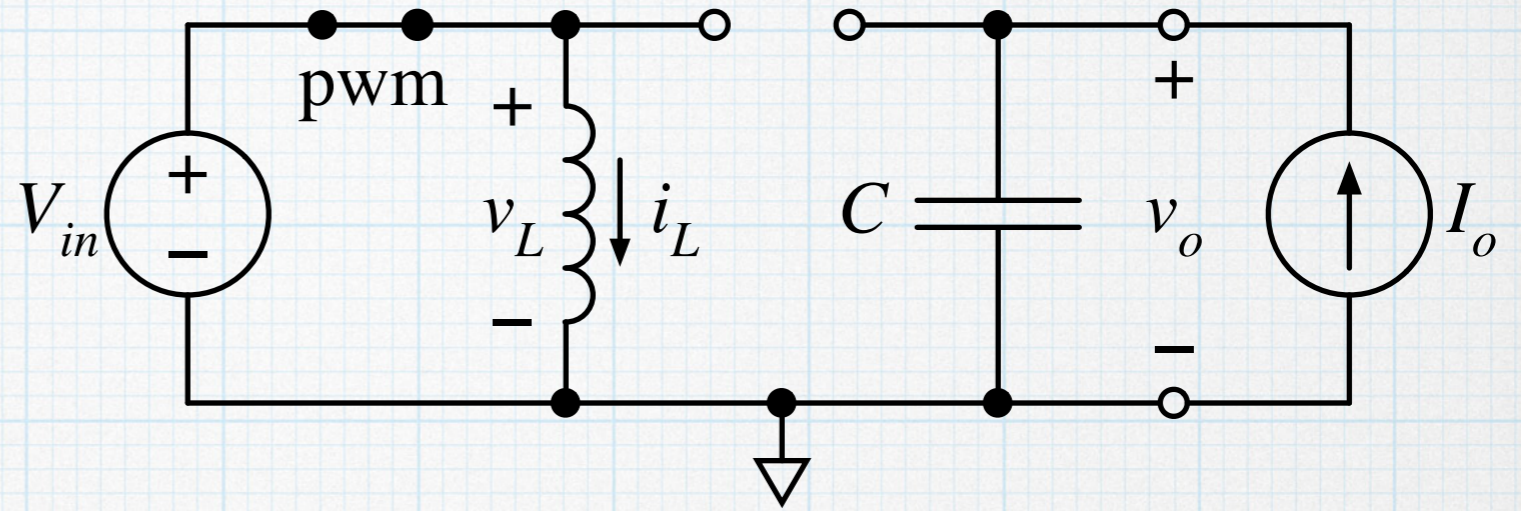
Note the change in the direction of the load current to be consistent with a negative output voltage.

The steady-state analysis is nearly identical to the boost and buck examples — we assume that the components are ideal and that the input and output voltages are constant, $V_{in} > 0$ and $v_o < 0$.

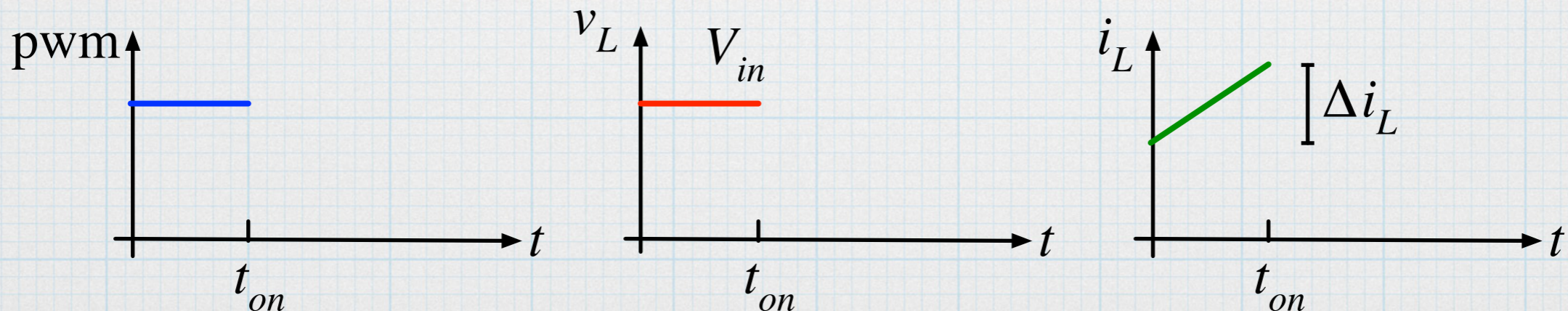
Invert – switch closed

$(0 < t < t_{on})$

The inductor current starts at some value, $i_L(0)$. With the switch closed:



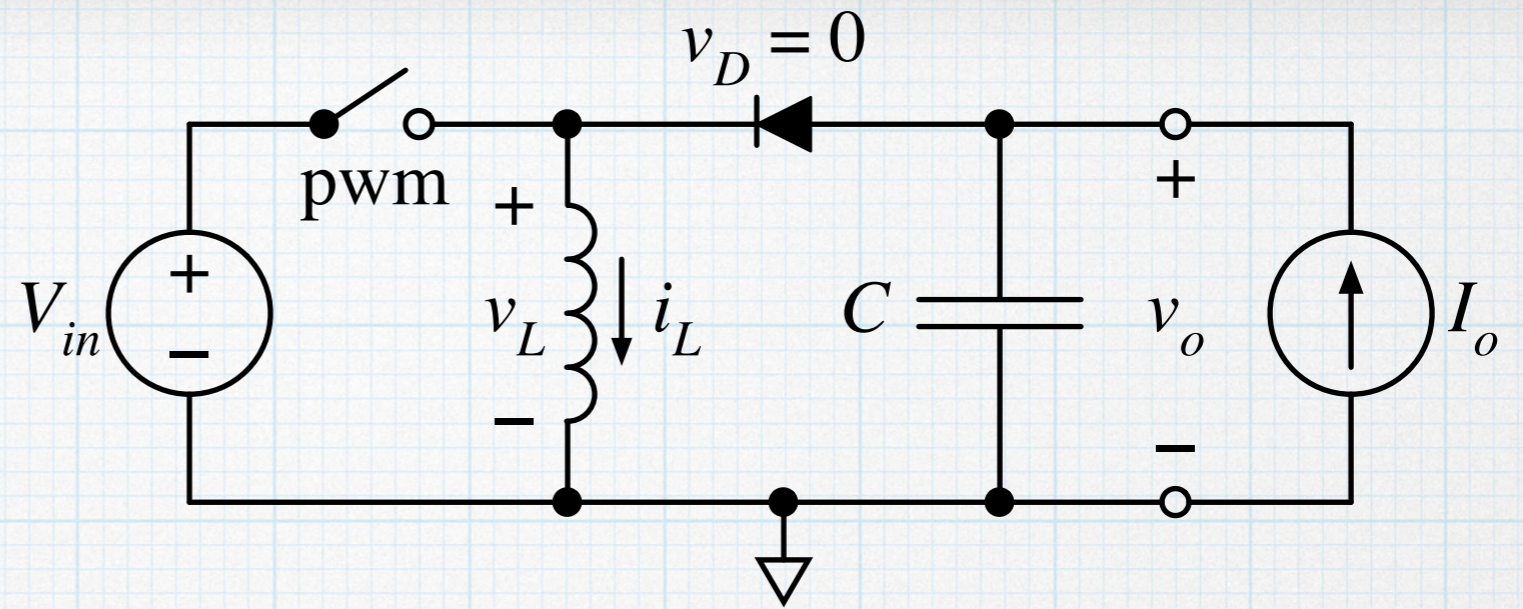
- The voltage across the inductor is $v_L = V_{in}$.
- The diode is reverse-biased ($v_D = v_o - V_{in}$ with $v_o < 0$), effectively disconnecting the input from the output.
- The inductor current increases with time: $i_L(t) = \frac{1}{L} V_{in} \cdot t + i_L(0)$
- The current increase during the “on” time is: $\Delta i'_L = \frac{V_{in}}{L} \cdot t_{on}$
- The inductor energy increases with time.



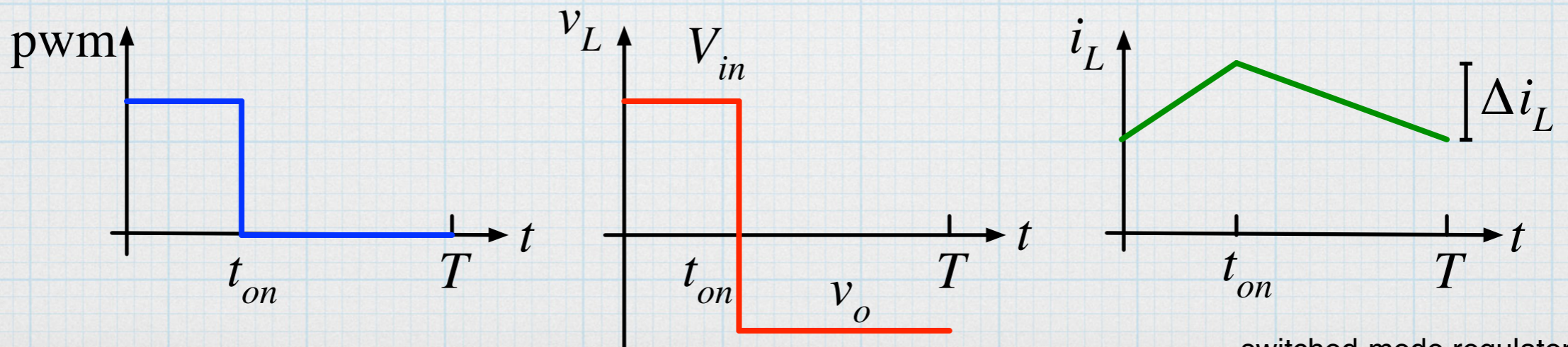
Invert - switch open

$(t_{on} < t < T)$

The inductor current starts at $i_L(t_{on})$. When the switch is open:



- The inductor current can no longer flow through the switch – it must flow through the diode. To turn on the diode, the voltage at upper end of the inductor must drop to v_o , which is negative. The inductor voltage becomes $v_L = v_o$.
- The inductor current decreases with time: $i_L(t) = \frac{v_o}{L} (t - t_{on}) + i_L(t_{on})$
- The current decrease during the off time is: $\Delta i_L'' = \frac{v_o}{L} (T - t_{on})$
- The inductor energy decreases with time.



It's the same story. The power flow must balance — energy is added to the inductor when the switch is closed, and that energy is transferred to the capacitor and load when the switch is open.

The magnitude of the current increase in the inductor must match the magnitude of the current decrease.

$$\frac{V_{in}}{L} t_{on} = - \frac{v_o}{L} (T - t_{on})$$
$$v_o = \frac{t_{on}}{t_{on} - T} V_{in} = - \frac{D}{1 - D} V_{in}$$

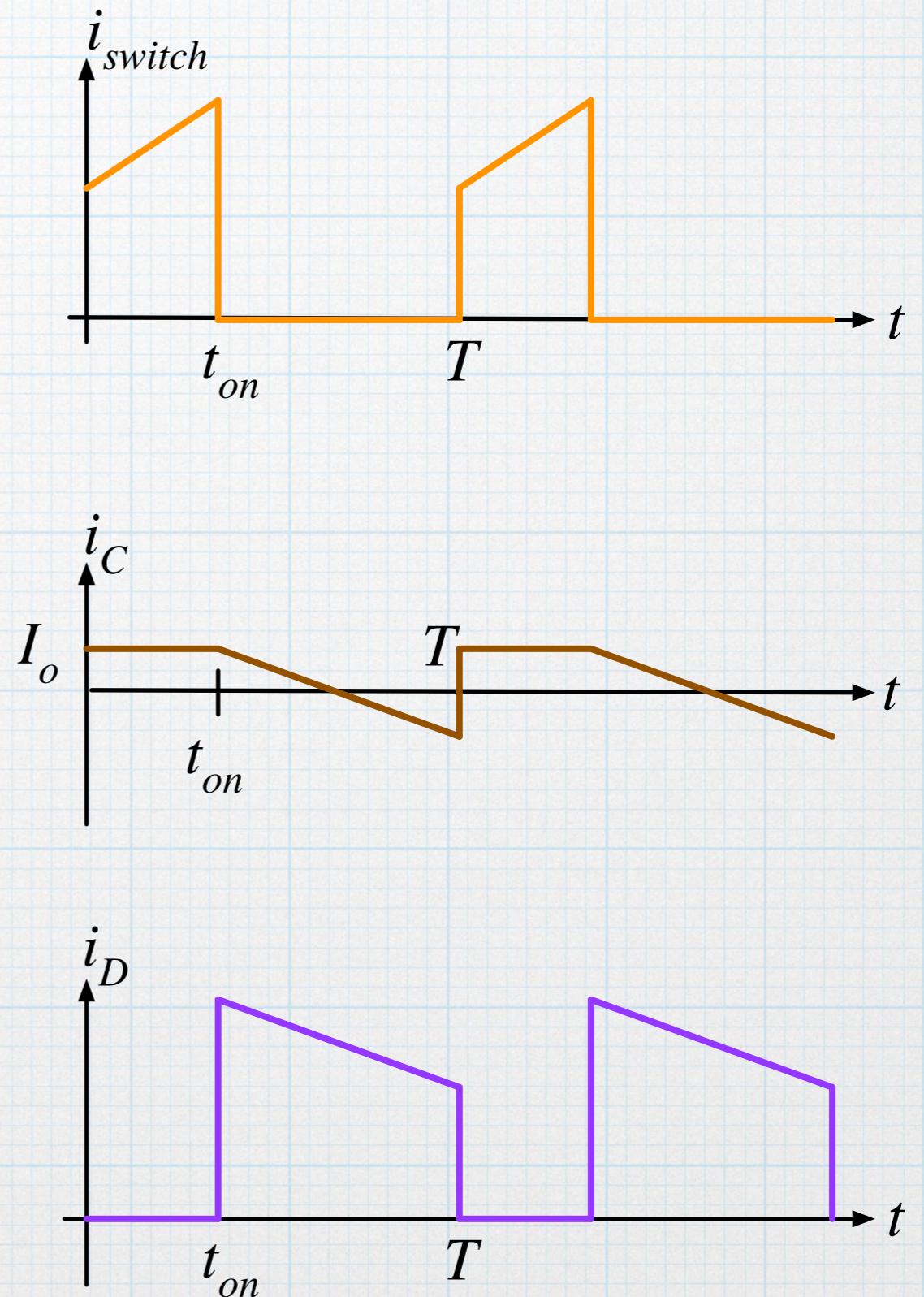
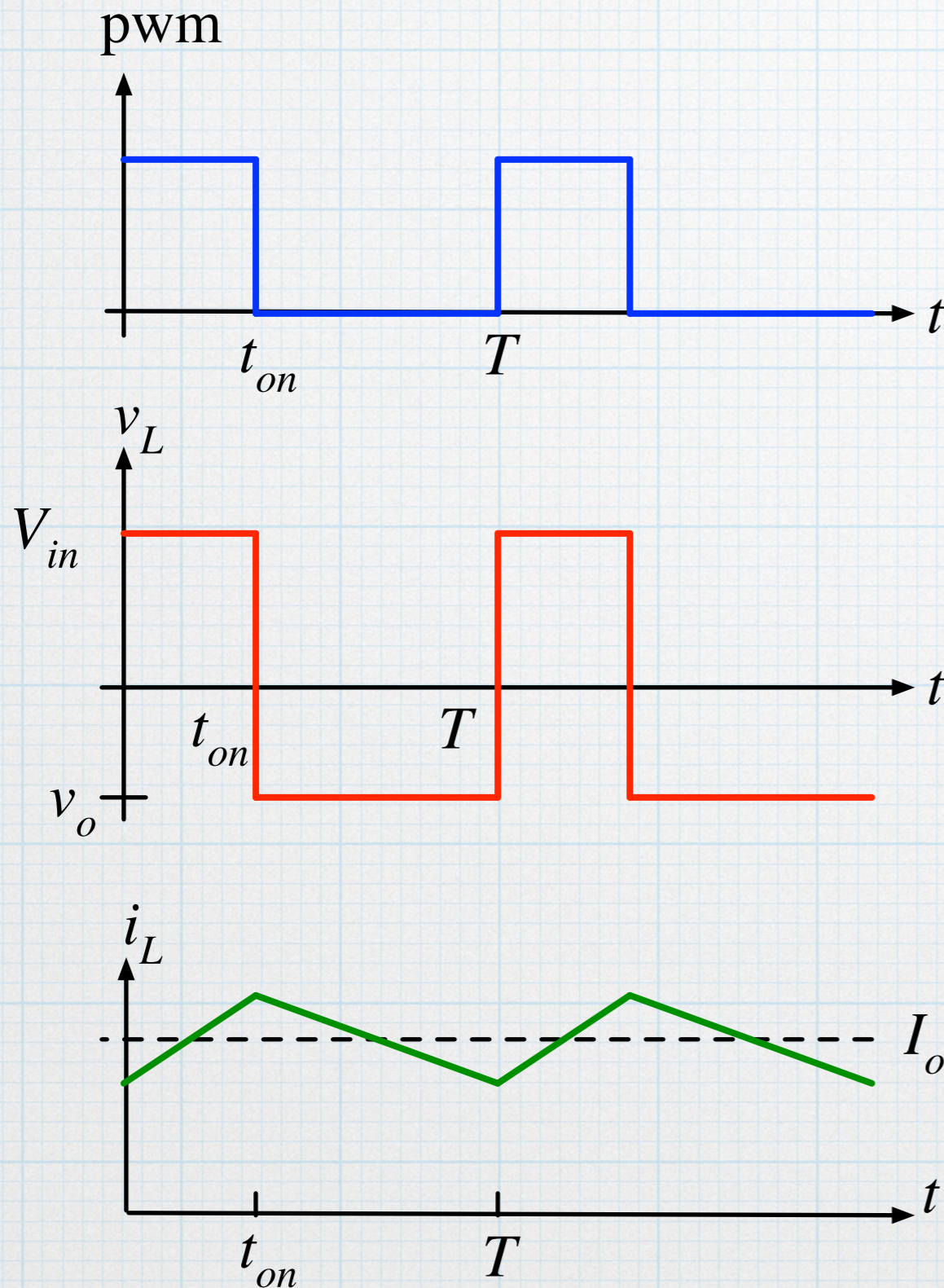
The output is always negative, but the magnitude can be bigger or smaller than V_{in} .

$$v_o = -V_{in} \text{ for } D = 0.5, \quad |v_o| < V_{in} \text{ for } D < 0.5, \quad |v_o| > V_{in} \text{ for } D > 0.5.$$

Note that during the on time, when the switch is closed, current flows from the source into the inductor, but with the diode off, there is no current flowing into the cap/load combination. During the on time, the capacitor must supply the entire load current, $i_C = I_o$. When the switch is off, no current flows from the source, but the inductor supplies current to the cap/load. In the inverting case, neither the source current nor the current to the load/cap is continuous.

inverting waveforms

In the inverting case, the current from the source is the same as the switch current.



We see that the three types of SMRs exhibit some similar characteristics. The various waveforms are nearly identical. The differences arise from the way that voltages are applied across the inductor in the open and closed switch configurations. In all cases, if we perfect components, there is perfect efficiency.

There are some differences: In the buck converter case, the current from the source is not continuous but the current to the capacitor-load combination is continuous. In the boost configuration, the source current is continuous, but the capacitor-load current is discontinuous. For the inverting circuit, both the source and capacitor/load currents are discontinuous.

We might also note that our assumption of absolutely constant output voltage, even with constant output current, cannot really be true. In all three cases, we see that capacitor current is ramping up and down during each switching cycle. If the current is changing, there must be non-zero dv_C/dt — the voltage must be changing, too. Thus, we expect some rippling of the output voltage. However, in theory the voltage ripple can be reduced to an arbitrarily small value by choosing a sufficiently large capacitor. A common mis-conception about switching regulators is that the ripple voltage will always be must worse than a linear regulator. In fact, that two can be equally steady with correct choice of components.