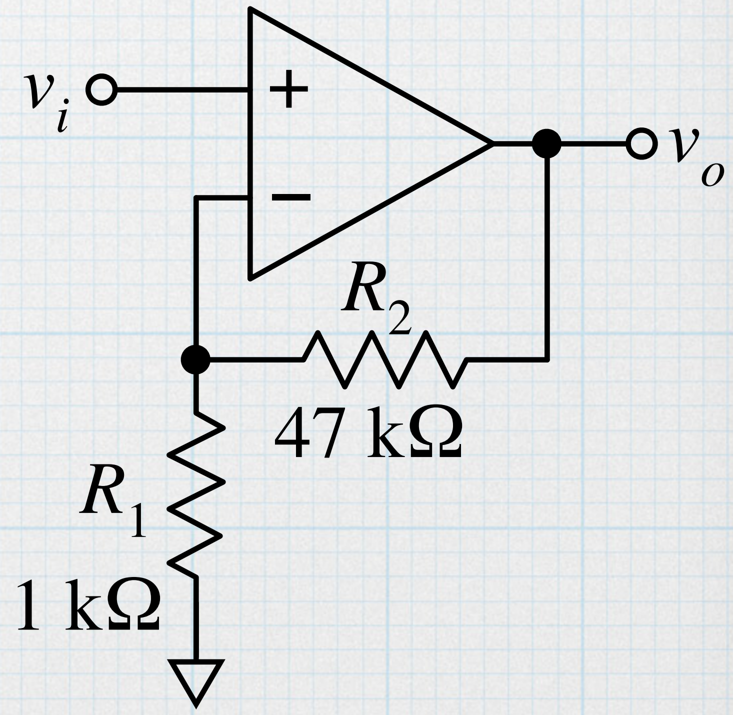
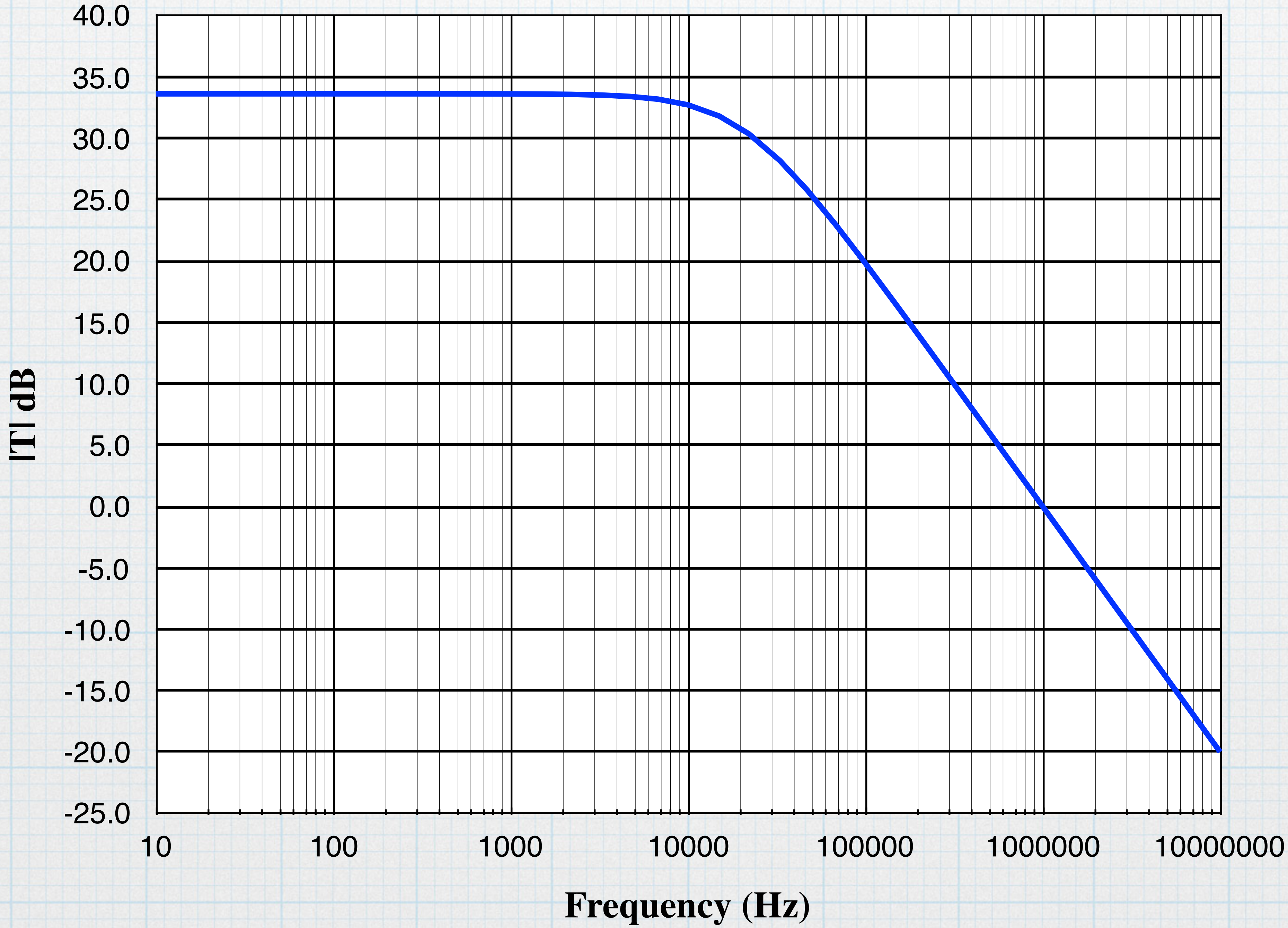
Bandwidth limits of op amps

Consider the following scenario: You need to amplify a signal that has a primary oscillation at 30 kHz. The signal is small — the amplitude around 20 mV. We can model the signal as a 30-kHz sinusoid with an amplitude of 20 mV. You would like to amplify with a gain of about 50 to produce an output on the order of 1 V in order to pass the signal along to the next stage in the system that is processing the signal. So you build a simple non-inverting circuit with an LM324 amp like that shown below. You have done this many times before so you expect no problems.

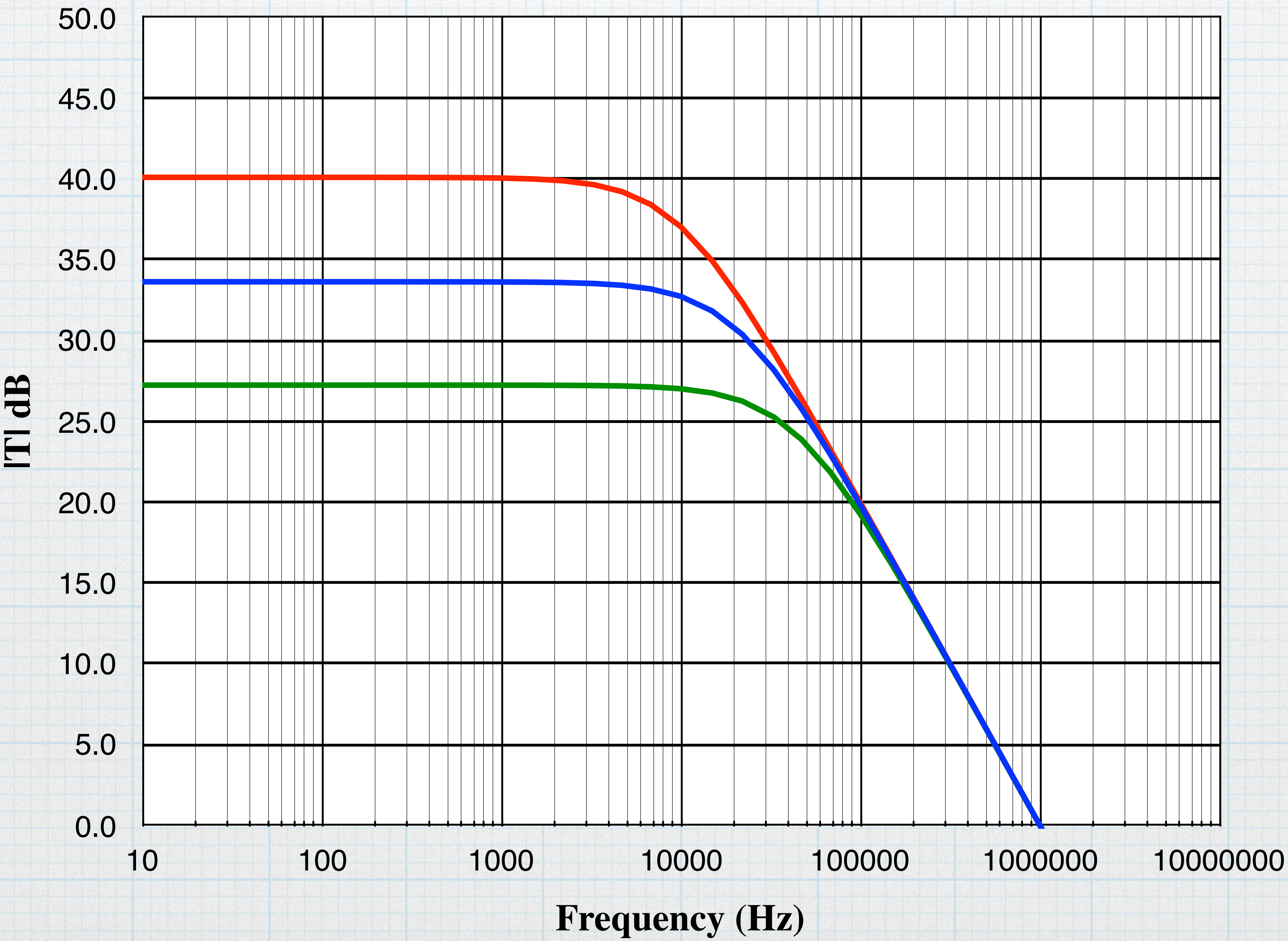
However, when you wire this up, and apply the 20-mV amplitude test signal, the output amplitude is not the expected 0.96 V, but only 0.55V. Hmmm. You check all the obvious things and nothing is amiss. Curious, you play a bit. You try decreasing the frequency to 15 KHz and find the the amplitude is 0.78 V. Closer, but still not right. You decrease all the way to 1 kHz and find the amplitude is then the expected 0.96 V. What is going on?



You go a step further and measure a complete frequency response, shown below. The closed-loop amp, with no external capacitors in sight is showing a definite low-pass response. The corner frequency is at about 20.8 kHz. This might *imply* that there is a 0.16-nF capacitor in parallel with R_2. But again, there is no possible capacitance there.



Trying to make more sense of this, you change the gain by changing R_2, first to 22 kΩ (low frequency gain = 23) and then to 100 kΩ (low frequency gain =101). For each new gain, you make complete frequency response — the three cases are plotted together below. (Red is $G = 101$, blue is $G = 48$, green is $G = 23$). When the gain was decreased, the corner moved to 43.5 kHz. When the gain was increased, the corner moved to 9.90 kHz.



After poking around some more and eliminating all other possible external causes that might introduce a low-pass response, you settle on the only conclusion remaining — the op-amp itself has an "intrinsic" low-pass frequency response. Somehow, the gain of the amp must go down at higher frequencies. This is definitely a modification of our ideal view of an op-amp.

We don't know the mechanism of the high-frequency roll off of the op-amp, but we can guess – there must be capacitor internal to the amp.

That is exactly what is going on. A certain amount of capacitance is purposely added to the op-amp circuitry by the designers to improve the *stability* of the amp. (We will examine the issue of amplifier stability soon enough.) The is the same capacitance that causes to the limited slew rate of the op amp.

The corner frequency of the closed-loop amp changes with the gain in a predictable fashion. This suggests that the low-pass behavior of the op amp itself follows a simple low-pass model, something like:

$$A \rightarrow A(s) = A_o \cdot \frac{\omega_{co}}{s + \omega_{co}}$$

where A_o is the gain at low frequencies and ω_{co} is the corner frequency of the open-loop gain. What will be the effect if we put a regular feedback loop around the amp? We can use feedback theory to see what is happening.

$$G = \frac{A}{1 + A\beta}$$

If A is frequency dependent, $A \rightarrow A(s)$, then G must also be frequency dependent

$$G(s) = \frac{A(s)}{1 + A(s)\beta}$$

Given the proposed low-pass open-loop gain $A(s) = A_o \cdot \frac{\omega_{co}}{s + \omega_{co}}$, we can insert it into the classic feedback formulat.

$$G(s) = \frac{A_o \cdot \frac{\omega_{co}}{s+\omega_{co}}}{1 + \left(A_o \cdot \frac{\omega_{co}}{s+\omega_{co}}\right)\beta} = \frac{A_o \cdot \omega_{co}}{s + \omega_{co} + A_o \beta \omega_{co}} = \frac{A_o \cdot \omega_{co}}{s + \omega_{co}\left(1 + A_o \beta\right)}$$

Indeed, $G(s)$, has a low-pass form with a corner frequency

$$\omega_{cc} = \omega_{co}\left(1 + A_o \beta\right) \quad \text{and} \quad G_o \omega_{cc} = A_o \omega_{co}$$

$$G_o = \frac{A_o \omega_{co}}{\omega_{cc}} = \frac{A_o}{1 + A_o \beta}$$

The effect of the simple feedback loop on a low-pass open-loop gain function characterized by A_o and ω_{co} is to lower the gain by $(1+A_o\beta)$ and extend corner frequency by $(1+A_o\beta)$. Very interesting. Of course, this is exactly what we were seeing in our little scenario.

It is worth repeating. An amp that has an open-loop low-pass response with gain A_o and corner frequency ω_{co}

$$A(s) = A_o \cdot \frac{\omega_{co}}{s + \omega_{co}}$$

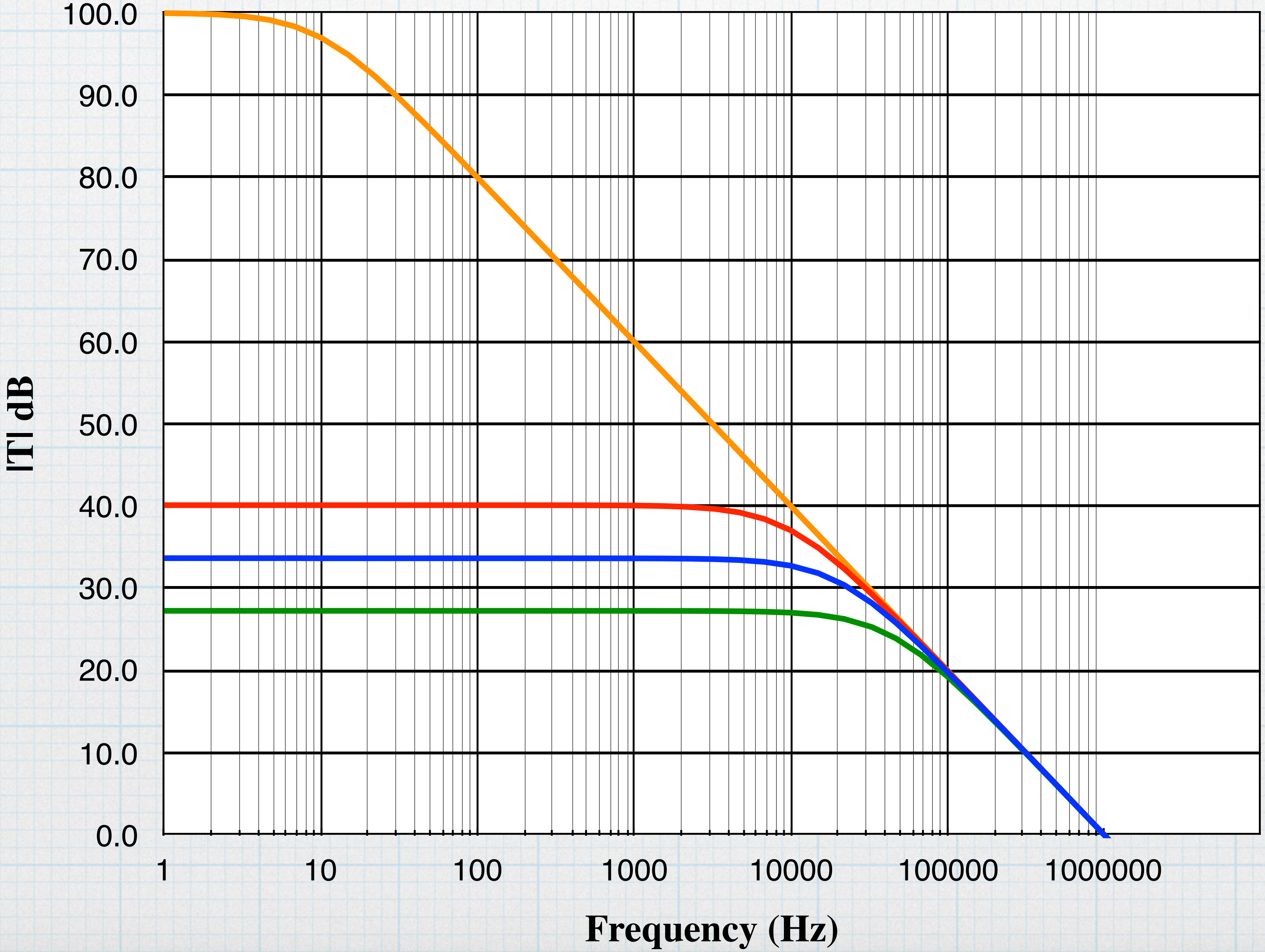
leads to another low-pass function when a feedback loop is applied,

$$G(s) = G_o \cdot \frac{\omega_{cc}}{s + \omega_{cc}}$$

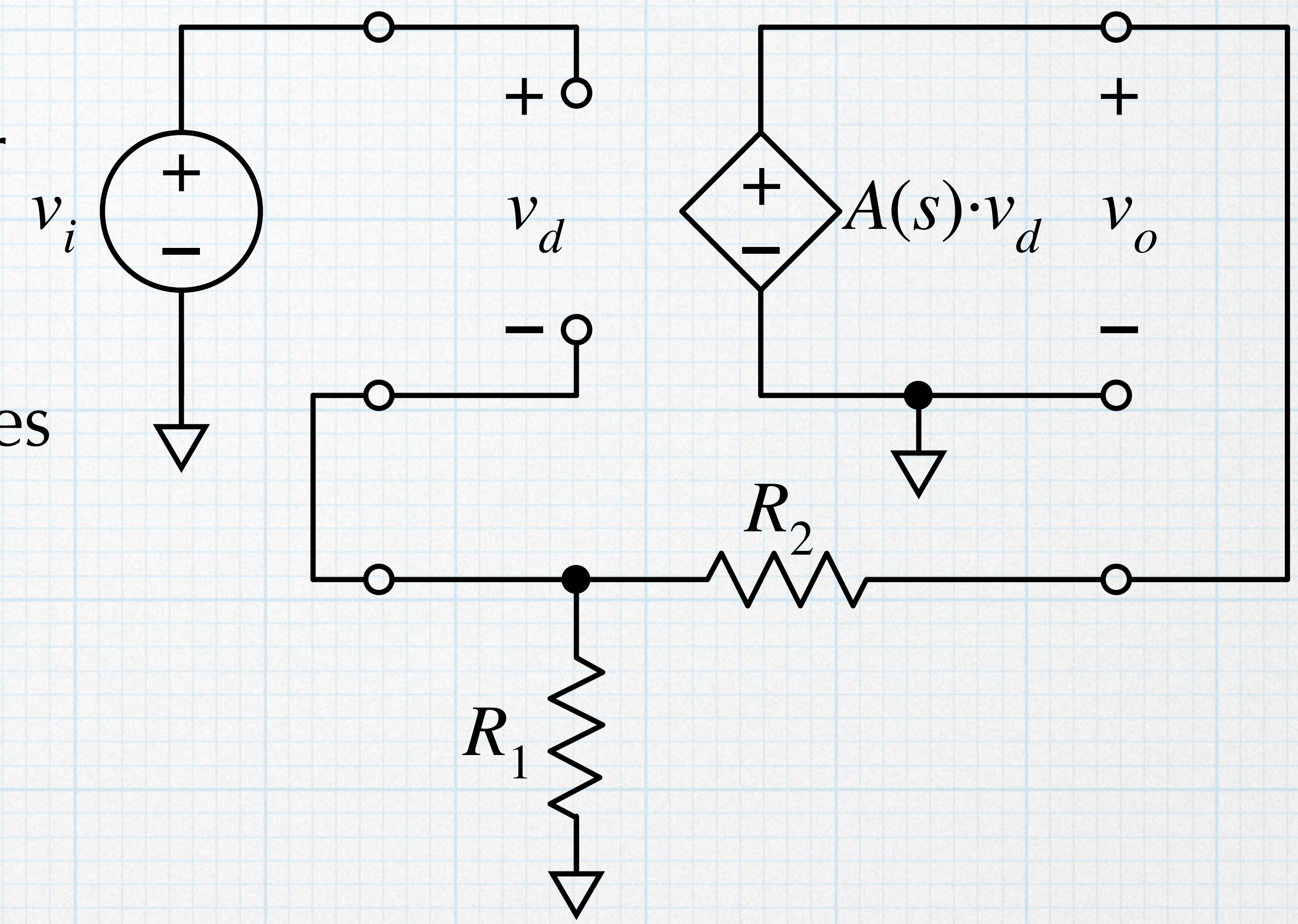
where $\omega_{cc} = \omega_{co}\left(1 + A_o\beta\right)$ and $G_o = \frac{A_o\omega_{co}}{\omega_{cc}} = \frac{A_o}{1 + A_o\beta}$.

The quantity $A_o\omega_{co}$ plays key real role in the analysis. Since both A_o and ω_{co} are properties of the amp, the product is a property is a the amp. Not surprisingly, it is called the gain-bandwidth of the amp. The units are those of frequency, usually expressed in hertz, although, as usual, we will work interchangeably between f and ω.

For an op amp, we expect the open-loop gain to be big, $A_o > 10^5$. However, the open-loop corner is surprisingly low, maybe 10 Hz less. (More on this later.) Using these numbers, the gain-bandwidth would be $A_o f_{co} = (10^5)(10\ \text{Hz}) = 1\ \text{MHz}$. This value is actually quite typical for many general purpose op amps. We can see the role the the open-loop gain-bandwidth plays if we plot the open-loop low-pass function together with the closed-loop functions of the previous plot.



We can also go at it with straight-forward circuit analysis. Consider a non-inverting amp, drawn using the two-port model. We have found the gain for this several times in the past.



$$G = \frac{v_o}{v_i} = \frac{1 + \frac{R_2}{R_1}}{\frac{1}{A}\left(1 + \frac{R_2}{R_1}\right) + 1}$$

To see the effect of the low-pass of the the op amp, we replace A in the above expression with its low-pass frequency-dependent form.

$$A \rightarrow A(s) = A_o \frac{\omega_{co}}{s + \omega_{co}}$$

$$G(s) = \frac{1 + \frac{R_2}{R_1}}{\frac{s + \omega_{co}}{A_o \omega_{co}}\left(1 + \frac{R_2}{R_1}\right) + 1}$$

Re-arrange a bit.

$$G(s) = \frac{A_o \omega_{co} \left(1 + \frac{R_2}{R_1}\right)}{\left(s + \omega_{co}\right)\left(1 + \frac{R_2}{R_1}\right) + A_o \omega_{co}} = \frac{A_o \omega_{co}}{s + \omega_{co}\left(1 + \frac{A_o}{1 + \frac{R_2}{R_1}}\right)}$$

Again, the closed-loop function is low pass

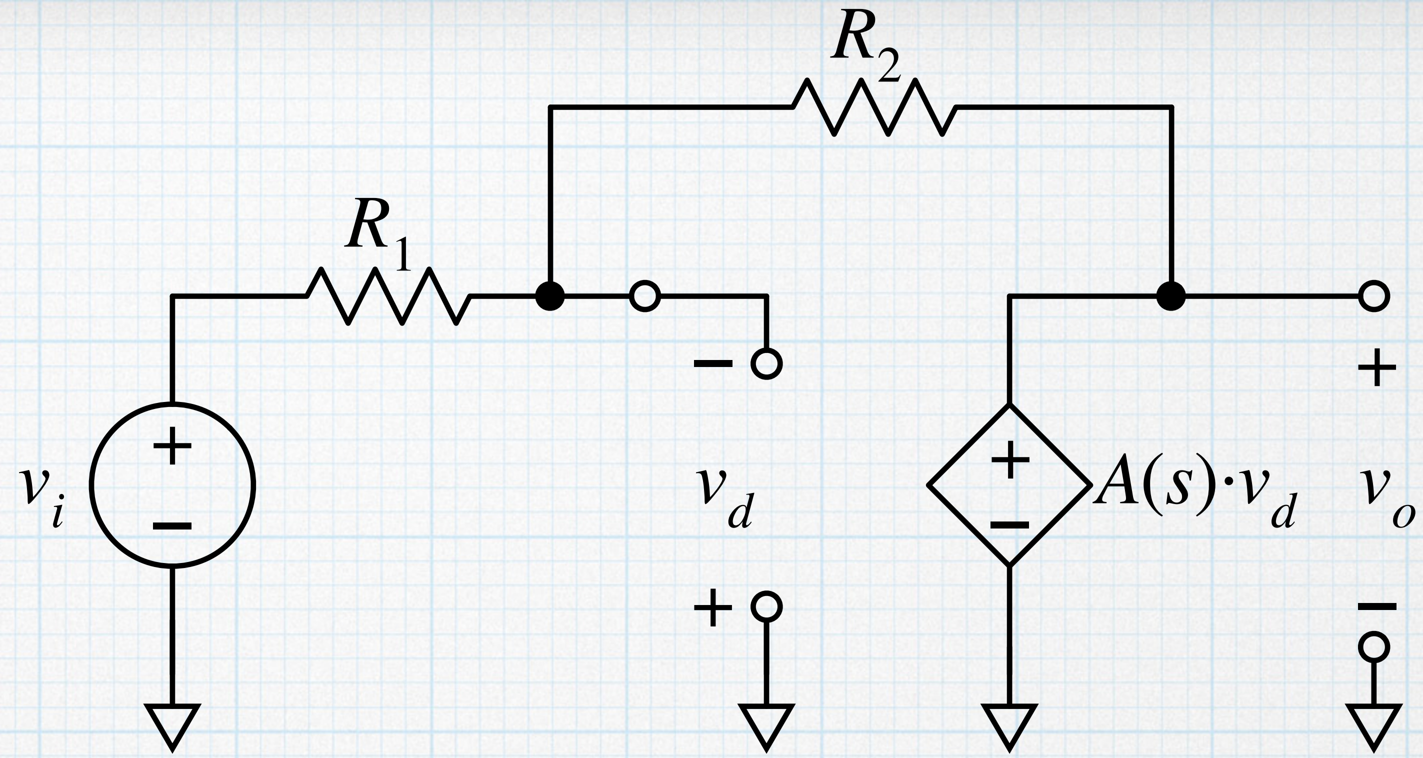
$$G(s) = G_o \cdot \frac{\omega_{cc}}{s + \omega_{cc}}$$

where $\omega_{cc} = \omega_{co}\left[1 + A_o\left(\frac{R_1}{R_1 + R_2}\right)\right]$

and $G_o = \frac{A_o \omega_{co}}{\omega_{cc}} = \frac{A_o}{1 + A_o\left(\frac{R_1}{R_1 + R_2}\right)} = \frac{\frac{R_1 + R_2}{R_1}}{1 + \frac{1}{A_o}\left(\frac{R_1 + R_2}{R_1}\right)} \approx 1 + \frac{R_2}{R_1}$

In the closed-loop case, the low-frequency gain is what we would expect for a non-inverting amp. The closed-loop corner frequency is extended to a value much higher than ω_{co}.

We see the exact same effects with an inverting amp. Again, to calculate the frequency response, we need to use the two-port model in an inverting configuration.



With a little bit of circuit analysis, we can find the expression for the closed-loop gain, including A, (Check it yourself.)

$$G = \frac{-\frac{R_2}{R_1}}{1 + \frac{1}{A}\left(\frac{R_2}{R_1} + 1\right)}$$

Inserting the frequency dependent form for $A \rightarrow A(s) = A_o \frac{\omega_{co}}{s + \omega_{co}}$

$$G = \frac{-\frac{R_2}{R_1}}{1 + \left(\frac{s + \omega_{co}}{A_o \omega_{co}}\right)\left(\frac{R_2}{R_1} + 1\right)}$$

Re-arranging:

$$G(s) = \frac{A_o \omega_{co}\left(-\frac{R_2}{R_1}\right)}{\left(s+\omega_{co}\right)\left(1+\frac{R_2}{R_1}\right)+A_o\omega_{co}} = \frac{-A_o\omega_{co}\left(\frac{R_2}{R_1+R_2}\right)}{s+\omega_{co}\left(1+\frac{A_o}{1+\frac{R_2}{R_1}}\right)}$$

Yet again, the closed-loop function is low pass

$$G(s) = G_o \cdot \frac{\omega_{cc}}{s+\omega_{cc}}$$

where $\omega_{cc} = \omega_{co}\left[1 + A_o\left(\frac{R_1}{R_1+R_2}\right)\right]$ (Same as with non-inverting.)

and $G_o = \frac{-A_o\omega_{co}}{\omega_{cc}} = \frac{-A_o\left(\frac{R_2}{R_1+R_2}\right)}{1+A_o\left(\frac{R_1}{R_1+R_2}\right)} = \frac{-\frac{R_2}{R_1+R_2}}{\frac{1}{A_o}+\left(\frac{R_1}{R_1+R_2}\right)} \approx -\frac{R_2}{R_1}$

Same story, the low-frequency gain is what we would expect for an inverting amp. The closed-loop corner frequency is extended to a value much higher than ω_{co}.

The importance of the gain-bandwidth

The key relationship for understanding the effect of the gain-bandwidth limit in amplifier circuits is

$$A_o \omega_{co} = G_o \omega_{cc}$$

This tells us that an amp has fixed amount of gain-bandwidth. When designing a closed-loop circuit with this amp, you can lots of gain but a with lower corner frequency (less bandwidth) or a higher corner frequency but lower gain. You cannot have both large gain and a large bandwidth (high corner frequency).

Going back to the original scenario using an LM324 op amp, which has a gain-bandwidth limit of 1 Mhz, we can now expect what we discovered by accident — with a feedback loop that creates a low-frequency gain of 48, the corresponding closed-loop low-pass response would have a corner at

$$f_{cc} = \frac{A_o f_{co}}{G_o} = \frac{1\,\text{MHz}}{48} = 20.8\,\text{kHz}.$$

Signals at frequencies above the corner will be corresponding reduced.