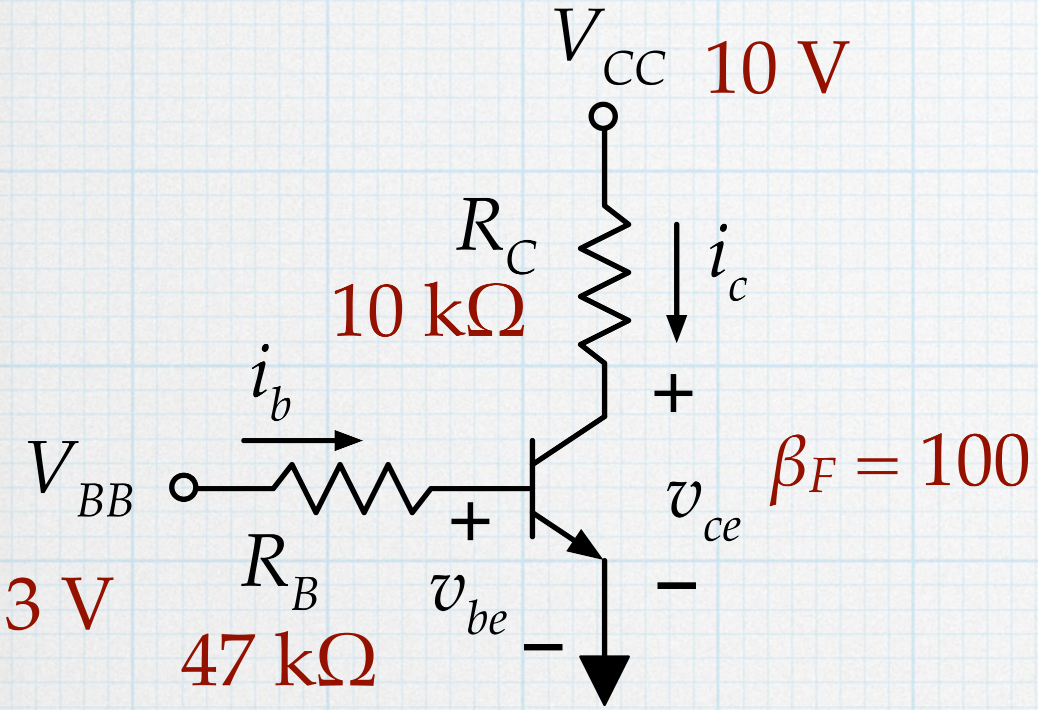
Example

Consider the circuit below. We have seen this one already. As before, assume that the BJT is on and in forward active operation.



$$i_B = \frac{V_{BB} - v_{BE}}{R_B} = \frac{3\,\text{V} - 0.7\,\text{V}}{47\,\text{k}\Omega} = 48.9\,\mu\text{A}$$

$$i_C = \beta_F i_B = (100)\left(48.9\,\mu\text{A}\right) = 4.89\,\text{mA}$$

$$v_{CE} = V_{CC} - i_C R_C$$

$$= 10\,\text{V} - (4.89\,\text{mA})\left(10\,\text{k}\Omega\right) = -\,38.9\,\text{V !!}$$

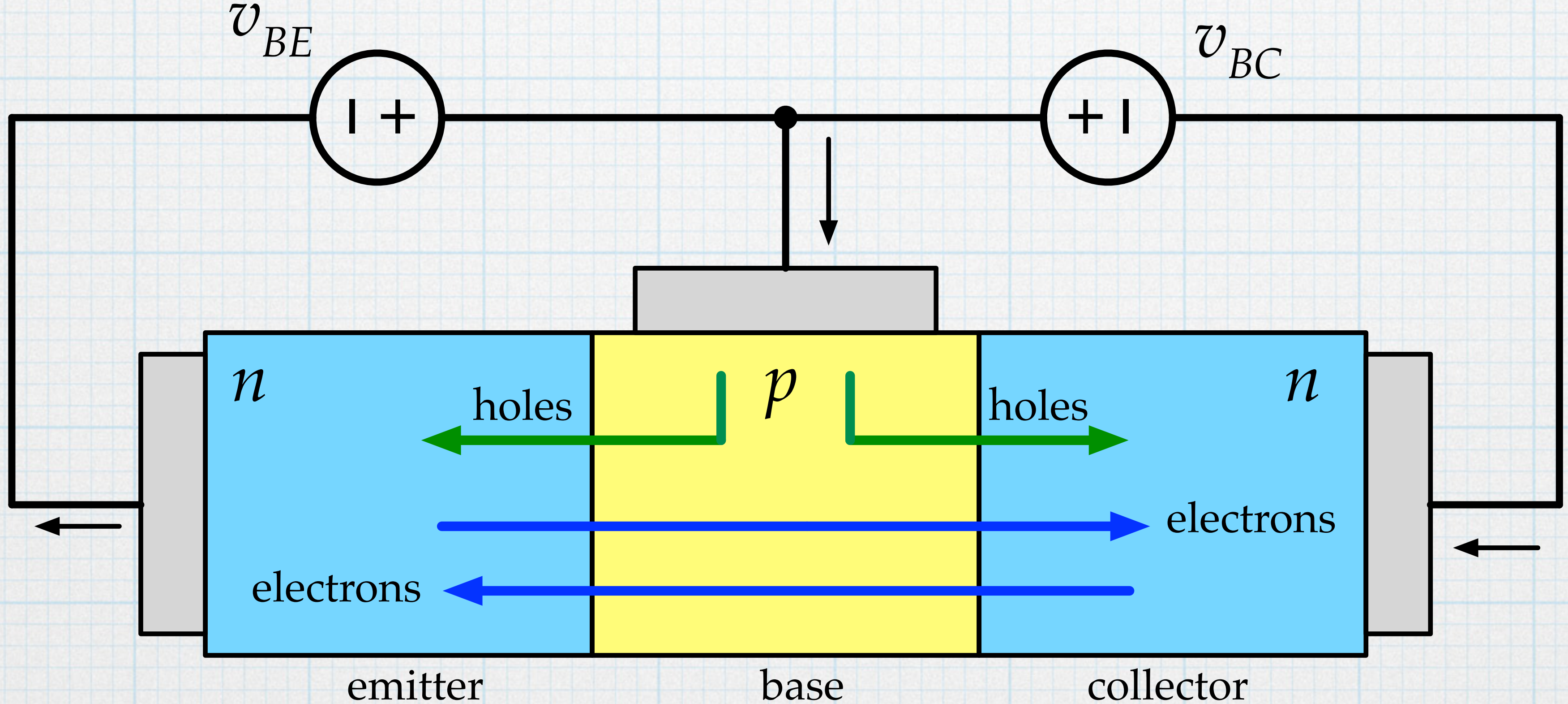
Red alert! Red Alert! Something is seriously wrong here — v_{CE} is extremely negative, which is not at all compatible with the B-C junction being reverse-biased. Furthermore, there is no way that we could get 39 V across any two points in the circuit when there is only one 10-V source and one 3-V source.

The problem lies in the assumption that the base-collector is reverse-biased. The collector current that we calculated is much too big.

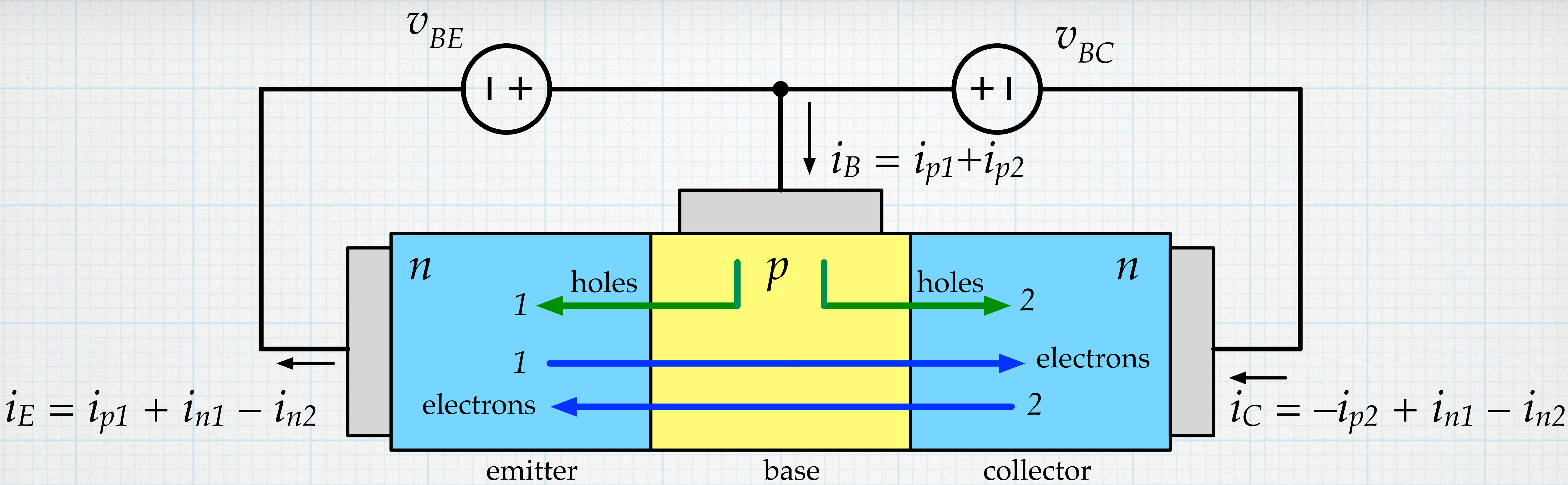
Saturation

When both junctions are forward-biased, the transistor is said to be in saturation. The B-E junction is still forward-biased, and electrons and holes are still injected across it. The electrons will still cross the base and travel into the collector, as we saw in the forward-active case.

But now, the base-collector is also forward-biased. Holes are being injected from base to collector. Also electrons are injected from collector to base. These electrons will cross the base and travel into the emitter.



In saturation, there are many more carriers flowing across the two junctions. The base current is increasing because of the additional injection of holes into the collector. The net collector current is decreasing because of the opposing electron flow from collector to emitter.



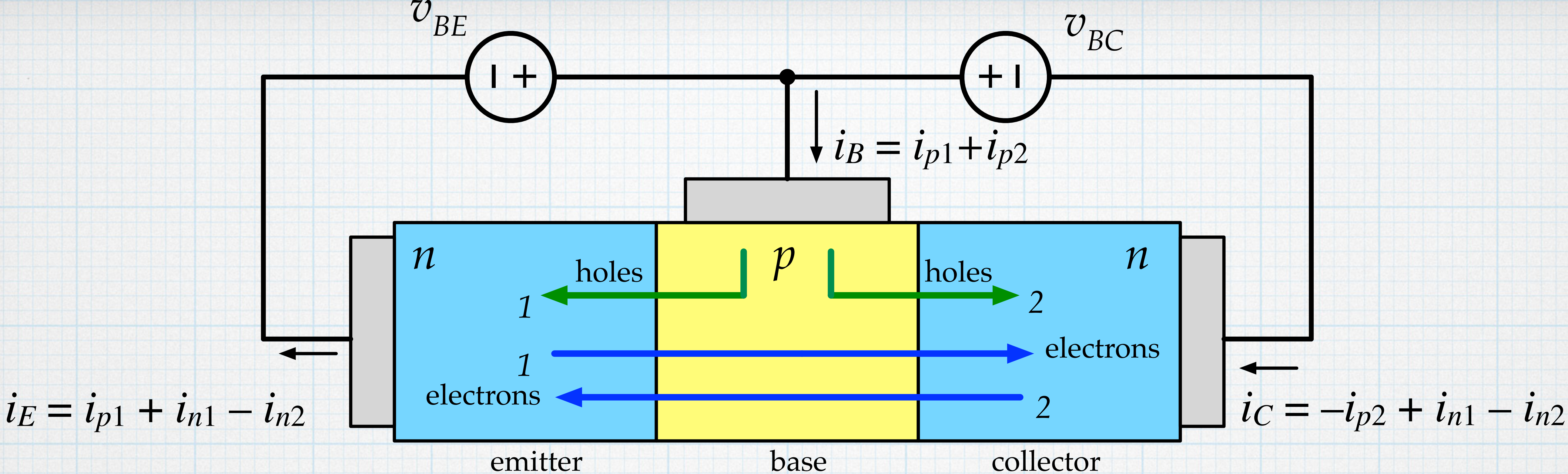
Now, we must consider diode-like currents crossing both junctions.

electrons injected emitter to base: $i_{n1} = I_{Sn1}\left[\exp\left(\frac{v_{BE}}{kT/q}\right) - 1\right] \approx I_{Sn1}\exp\left(\frac{v_{BE}}{kT/q}\right)$

holes injected base to emitter: $i_{p1} = I_{Sp1}\left[\exp\left(\frac{v_{BE}}{kT/q}\right) - 1\right] \approx I_{Sp1}\exp\left(\frac{v_{BE}}{kT/q}\right)$

electrons injected collector to base: $i_{n2} = I_{Sn2}\left[\exp\left(\frac{v_{BC}}{kT/q}\right) - 1\right] \approx I_{Sn2}\exp\left(\frac{v_{BC}}{kT/q}\right)$

holes injected base to collector: $i_{p2} = I_{Sp2}\left[\exp\left(\frac{v_{BC}}{kT/q}\right) - 1\right] \approx I_{Sp2}\exp\left(\frac{v_{BC}}{kT/q}\right)$



We can express the terminal currents in terms of the four electron and hole currents.

base current (all holes): $$i_B = I_{Sp1}\exp\left(\frac{v_{BE}}{kT/_q}\right) + I_{Sp2}\exp\left(\frac{v_{BC}}{kT/_q}\right)$$

collector current: $$i_C = I_{Sn1}\exp\left(\frac{v_{BE}}{kT/_q}\right) - I_{Sn2}\exp\left(\frac{v_{BC}}{kT/_q}\right) - I_{Sp2}\exp\left(\frac{v_{BC}}{kT/_q}\right)$$

emitter current: $$i_E = I_{Sn1}\exp\left(\frac{v_{BE}}{kT/_q}\right) - I_{Sn2}\exp\left(\frac{v_{BC}}{kT/_q}\right) + I_{Sp1}\exp\left(\frac{v_{BE}}{kT/_q}\right)$$

Wow - quite a mess. Note that i_B has increased (extra holes injected out of base) and i_C has decreased (electron currents are partially canceling), so $i_C < \beta_F \cdot i_B$. We don't even have that simplification in this case!

It would appear that trying to solve a circuit with a transistor that is in saturation would a hopeless endeavor — and nearly is. (Thank goodness for SPICE.) Our only hope is if there is a simplifying approximation that can be used for a BJT in saturation, and fortunately, there is one. To get to it will require a little math and a bit of hand-waving, but the simplification will be surprisingly "simple". Start with full equations for i_C and i_B.

$$i_B = I_{Sp1} \exp\left(\frac{v_{BE}}{kT/q}\right) + I_{Sp2} \exp\left(\frac{v_{BC}}{kT/q}\right)$$

$$i_C = I_{Sn1} \exp\left(\frac{v_{BE}}{kT/q}\right) - I_{Sn2} \exp\left(\frac{v_{BC}}{kT/q}\right) - I_{Sp2} \exp\left(\frac{v_{BC}}{kT/q}\right)$$

First we note that v_{BC} can be written in terms of v_{BE} and v_{CE}:

$$v_{BC} = v_{BE} - v_{CE}.$$

Secondly, we make use of a property that is true for all npn BJTs, namely that $I_{Sn1} = I_{Sn2}$. We will not to prove or justify this here — we will save it for EE 332. But in knowing this, we no longer need to distinguish between the two junctions, and we can rename the electron scale currents as just I_{Sn}.

Using the two observations from the previous slide:

$$i_B = I_{Sp1} \exp\left(\frac{v_{BE}}{kT/_q}\right) + I_{Sp2} \exp\left(\frac{v_{BE} - v_{CE}}{kT/_q}\right)$$

$$i_C = I_{Sn} \exp\left(\frac{v_{BE}}{kT/_q}\right) - I_{Sn} \exp\left(\frac{v_{BE} - v_{CE}}{kT/_q}\right) - I_{Sp2} \exp\left(\frac{v_{BE} - v_{CE}}{kT/_q}\right)$$

Collecting terms and re-writing:

$$i_B = I_{Sp1} \exp\left(\frac{v_{BE}}{kT/_q}\right) \left[1 + \frac{I_{Sp2}}{I_{Sp1}} \exp\left(\frac{-v_{CE}}{kT/_q}\right)\right]$$

$$i_C = I_{Sn} \exp\left(\frac{v_{BE}}{kT/_q}\right) \left\{1 - \left[1 + \frac{I_{Sp2}}{I_{Sn}}\right] \exp\left(\frac{-v_{CE}}{kT/_q}\right)\right\}$$

(You should check the math for yourself.)

Now take the ratio of the two functions:

$$\frac{i_C}{i_B} = \frac{I_{Sn}\exp\left(\frac{v_{BE}}{kT/q}\right)\left\{1 - \left[1 + \frac{I_{Sp2}}{I_{Sn}}\right]\exp\left(\frac{-v_{CE}}{kT/q}\right)\right\}}{I_{Sp1}\exp\left(\frac{v_{BE}}{kT/q}\right)\left[1 + \frac{I_{Sp2}}{I_{Sp1}}\exp\left(\frac{-v_{CE}}{kT/q}\right)\right]}$$

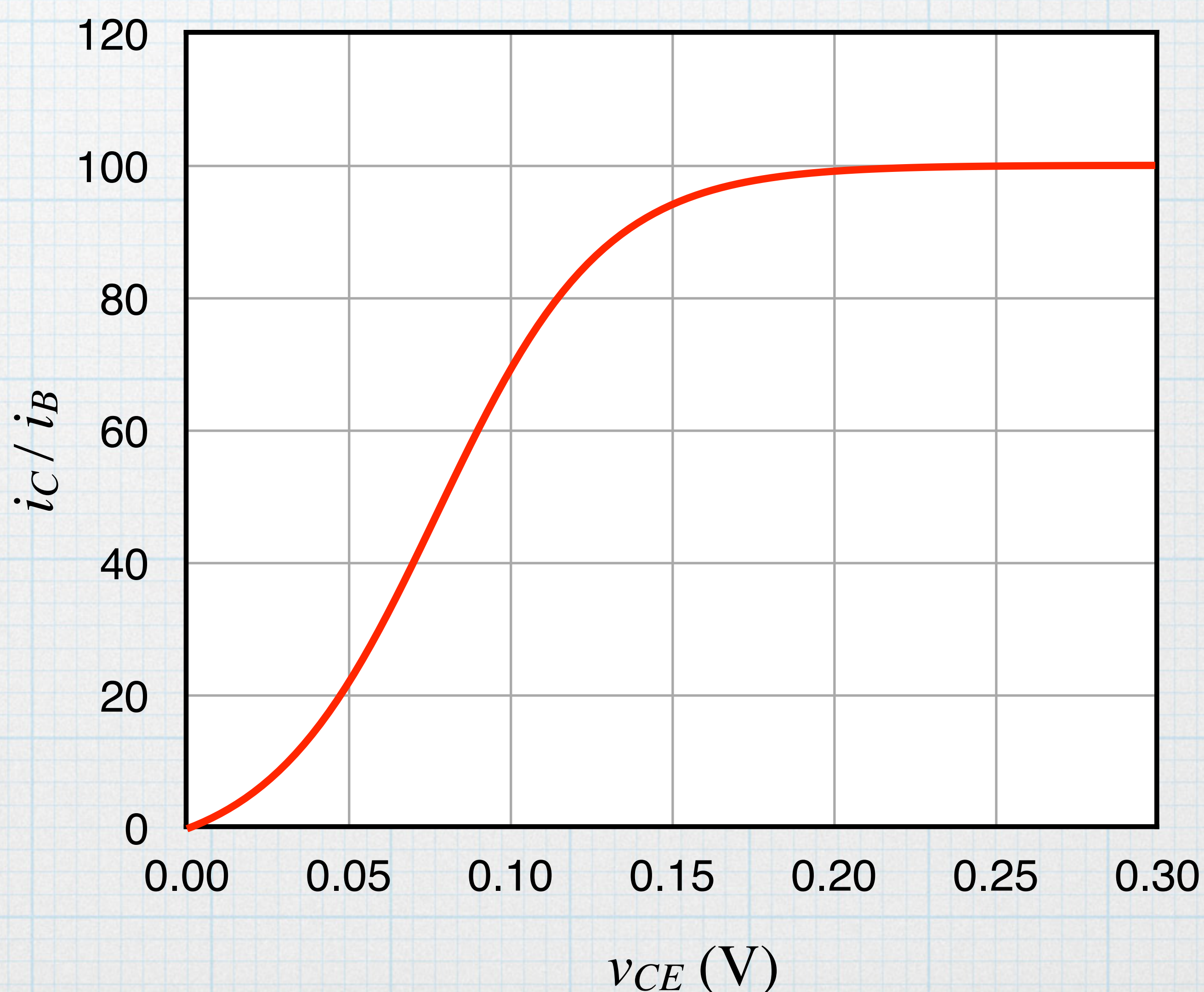
$$\frac{i_C}{i_B} = \frac{I_{Sn}}{I_{Sp1}}\left[\frac{1 - \left[1 + \frac{I_{Sp2}}{I_{Sn}}\right]\exp\left(\frac{-v_{CE}}{kT/q}\right)}{1 + \frac{I_{Sp2}}{I_{Sp1}}\exp\left(\frac{-v_{CE}}{kT/q}\right)}\right]$$

Of course, I_{sn}/I_{Sp1} is simply β_F. So this function is telling us how the forward current gain varies with v_{CE}. We already know that when v_{CE} is positive and "big" — meaning that the base-collector junction is reverse-biased — the current gain is a constant β_F. The function above agrees with that — when v_{CE} is many times bigger than kT/q, the exponential terms top and bottom go to zero and the entire function reduces to

$$\frac{i_C}{i_B} = \frac{I_{Sn}}{I_{Sp1}} = \beta_F,$$ which is just the forward-active mode.

So we really only need to investigate the situation when v_{CE} is on the order of kT/q. We can make a plot of the function for $0 < v_{CE} < \approx 10 \cdot kT/q$ to see how it behaves. In order to make a quantitive plot, we need to pick some values for the scale current ratios. Out a sense of consistency, we can choose $I_{Sn} / I_{Sp1} (= \beta_F) = 100$. In all practical BJTs, the the ratio of I_{Sn} to I_{Sp2} is much smaller — a typical value might be 5. (Again, we will save the explanation for why this is so for EE 332 and other classes.) That makes the ratio $I_{Sp2} / I_{Sp1} = 20$. The function and the corresponding plot are shown below.

$$\frac{i_C}{i_B} = 100 \cdot \left[\frac{1 - 1.05 \cdot \exp\left(\frac{-v_{CE}}{kT/q} \right)}{1 + 20 \cdot \exp\left(\frac{-v_{CE}}{kT/q} \right)} \right]$$



We see that variation in the currents occurs only when $v_{CE} \leq \approx 200$ mV. We can make this the starting assumption for saturation — if the BJT is in saturation, v_{CE} will be quite small.

For the BJT in saturation, we can use an approach much like the one we use for a forward-biased diode or base-emitter junction. With the diode, we decided that rather than struggling with exact the exponential relationship, we could use the approximation that forward voltage for a silicon p-n junction is always right around 0.7 V and make that the starting point for a simplified analysis. The choice of 0.7 V as the standard approximation was based on our observation of many different forward-biased junctions.

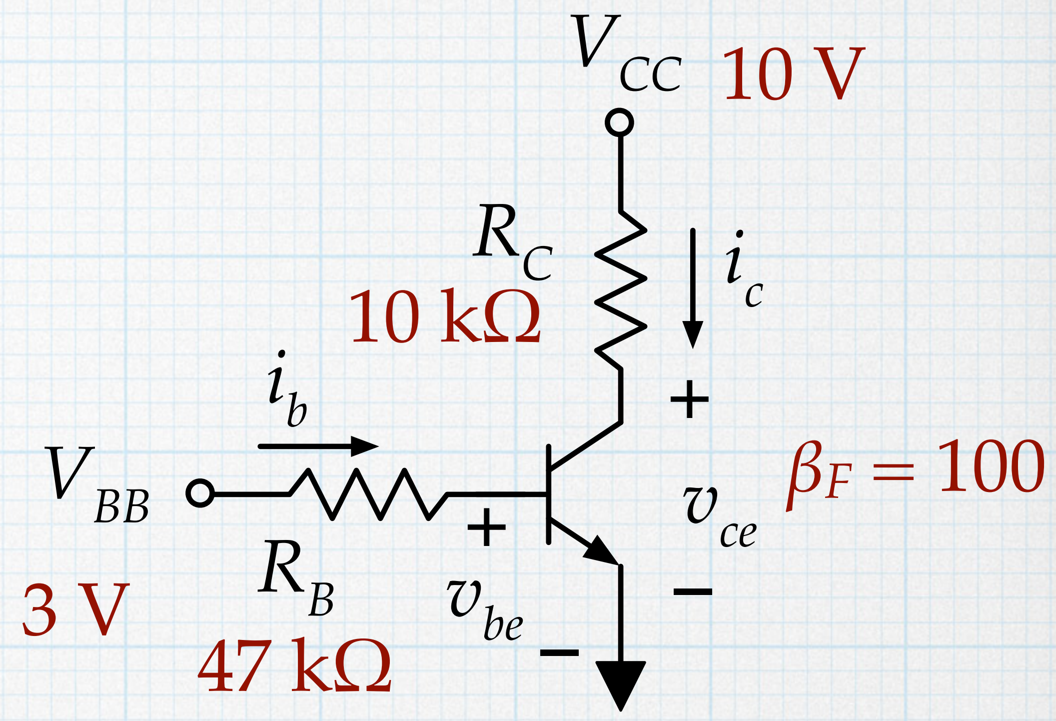
In the same way, we can start with our observation of the BJT as it goes into saturation — namely that the collector must be less than 0.2 V in saturation — and use that as the starting point for an approximate analysis for saturation. For simplicity, we will choose the saturated value of v_{CE} to be $v_{CE}\,(\text{sat}) = 0.2$ V. If we went to the lab and actually measured v_{CE} for a number BJTs in saturation, we would probably find that the saturated v_{CE} is typically smaller than 0.2 V — it might be 0.15 V or 0.1 V or even 50 mV. But in trying to argue that 0.15 V or 0.1 V would be a better choice for the saturation voltage, we would be splitting hairs. In *choosing* a value for $v_{CE}\,(\text{sat})$, we are making approximation, and the accuracy of the approximation will have little dependence on whether we choose 0.2 V or 0.15 V or 0.1 V. The important thing is to do choose a value and use it. For EE 230, we will use 0.2 V.

BJT in saturation

If you think that a BJT in a circuit is operating in saturation mode (vCE is small), the approach to solving the circuit is:

1. Assume that the base-emitter voltage is $v_{BE} = 0.7$ V. (Same approximation that we have been using all along for diodes and BJTs in forward active.)
2. Assume that the collector-emitter is $v_{CE} = 0.2$ V.
3. Use the usual KCL, KVL and other 201 techniques to find i_B, i_C, i_E, and any other required quantities. Note that, in saturation, $i_C \neq \beta_F i_B$!! The transistor can either be in saturation or forward-active — not both. You cannot use the rules from one mode to analyze a BJT that is in the other mode.
4. When finished, you must check to confirm that the BJT is in saturation. The check is that i_C must be less than $\beta_F i_B$ — $i_C < \beta_F i_B$. In other words it must be on the "curvy" part of the graph on slide 15. The confirmation is important!

Example (re-visited)



Now, we can re-consider the circuit from opening example. We know that it is not in forward active, so it must be in saturation. Start by finding i_B — the calculation is unchanged, because our assumptions about the base-emitter junction are the same for saturation as for forward active.

$$i_B = \frac{V_{BB} - v_{BE}}{R_B} = \frac{3\,\text{V} - 0.7\,\text{V}}{47\,\text{k}\Omega} = 48.9\,\mu\text{A}$$

Using the approximation for saturation: $v_{CE} = 0.2$ V,

$$i_C = \frac{V_{CC} - v_{CE}}{R_C} = \frac{10\,\text{V} - 0.2\,\text{V}}{10\,\text{k}\Omega} = 0.98\,\text{mA}$$

Given our initial bad experience when we guessed forward-active we can be fairly certain of saturation, we can still check and confirm:

$$\frac{i_C}{i_B} = \frac{0.98\,\text{mA}}{0.049\,\text{mA}} = 20 < \beta_F$$

The calculation is quite simple. The approximation for saturation helps us avoid a lot of nasty math.

Saturation

Now, when analyzing BJTs, we are faced with having to make a choice as to the mode of the BJT: forward-active or saturation. We can always guess. Guessing is fine because we can analyze the circuit based on the guess and then check to see of the results are consistent with the guess.

However, it more efficient if we have some means other than a coin flip for guessing. To start, let us look at what saturation means — essentially, the BJT "runs out of v_{CE}". Consider the KVL around the collector-emitter loop of a simple BJT circuit (like in the example) but operating in the forward-active mode.

$$V_{CC} - i_C R_C - v_{CE} = 0.$$

The available power supply voltage is divided between the resistor voltage and v_{CE}. If the resistor voltage is big, then v_{CE} must be small. If we think about increasing $i_C R_C$, then v_{CE} must decrease correspondingly. In the extreme, it would head towards zero. But it can never get there, because the transistor responds by going into saturation — the base-collector junction becomes forward biased. So we see that it is v_{CE} that "saturates" — it can't get any smaller and the BJT changes mode accordingly.

Hints

If we have solved enough forward-active practice problems, we should have some feel for typical numbers. Armed with that intuition, we can look for hints in the circuit that may indicate that a BJT is actually in saturation. In saturation, the collector-to-emitter is getting "squeezed", so we look for situations where the voltage on resistive elements in the collector-emitter loop become "big-ish". Watch for

- Any large i_C. (Results in making $i_C R_C$ big.)
- Large R_C. (Makes $i_C R_C$ big.)
- Small V_{CC}. (Reduces the available voltage range.)
- Large V_{BB}. (Increases i_B, which increases i_C in FA.)
- Small R_B. (Increases i_B, which increases i_C in FA.)
- Large β_F. (Increases i_C in FA.)

The effect of emitter resistors is a bit murkier. Even though they add another voltage drop in the CE loop, they also reduce base current.

If you take more electronics classes, you will learn how to design transistor circuits without using resistors. But even then, you must be on the lookout for the transistor going into saturation.