Diffusion example 1

A constant-source boron diffusion is performed into an *n*-type silicon wafer. The diffusion temperature is 1050°C and the diffusion time is 1 hour. Assume that the surface concentration is limited by the solid-solubility limit. If the *n*-type background doping of the silicon is 5x10^{16} cm^{-3}, find the junction depth and the boron dose.

Start by finding *Dt* for this diffusion. From the data for boron

$$D = D_o \exp\left(-\frac{E_A}{kT}\right)$$

$$= \left(1.0\,\frac{\mathrm{cm}^2}{\mathrm{s}}\right)\exp\left[-\frac{3.5\,\mathrm{eV}}{\left(8.617\times10^{-5}\,\frac{\mathrm{eV}}{\mathrm{K}}\right)\left(1323\,\mathrm{K}\right)}\right]$$

$$= 4.64\times10^{-14}\,\frac{\mathrm{cm}^2}{\mathrm{s}}$$

$$Dt = \left(4.64\times10^{-14}\,\frac{\mathrm{cm}^2}{\mathrm{s}}\right)\left(3600\,\mathrm{s}\right) = 1.67\times10^{-10}\,\mathrm{cm}^2$$

Example 1 (cont.)

Next, determine the constant surface concentration. In this case, it will be equal to the solid-solubility limit at 1050°C. Interpolating from the table for boron,

$$N(0) = N_s \approx 2.4 \times 10^{20}\,\mathrm{cm}^{-3}.$$

Next, apply the equation that was derived for the junction depth in a constant-source diffusion:

$$x_j = 2\sqrt{Dt}\left[\mathrm{erf}^{-1}\left(1 - \frac{N_B}{N_s}\right)\right]$$

With $1 - \dfrac{N_B}{N_s} = 1 - \dfrac{5 \times 10^{16}\,\mathrm{cm}^{-3}}{2.4 \times 10^{20}\,\mathrm{cm}^{-3}} = 0.999792.$

The table of error function values (next page) shows that

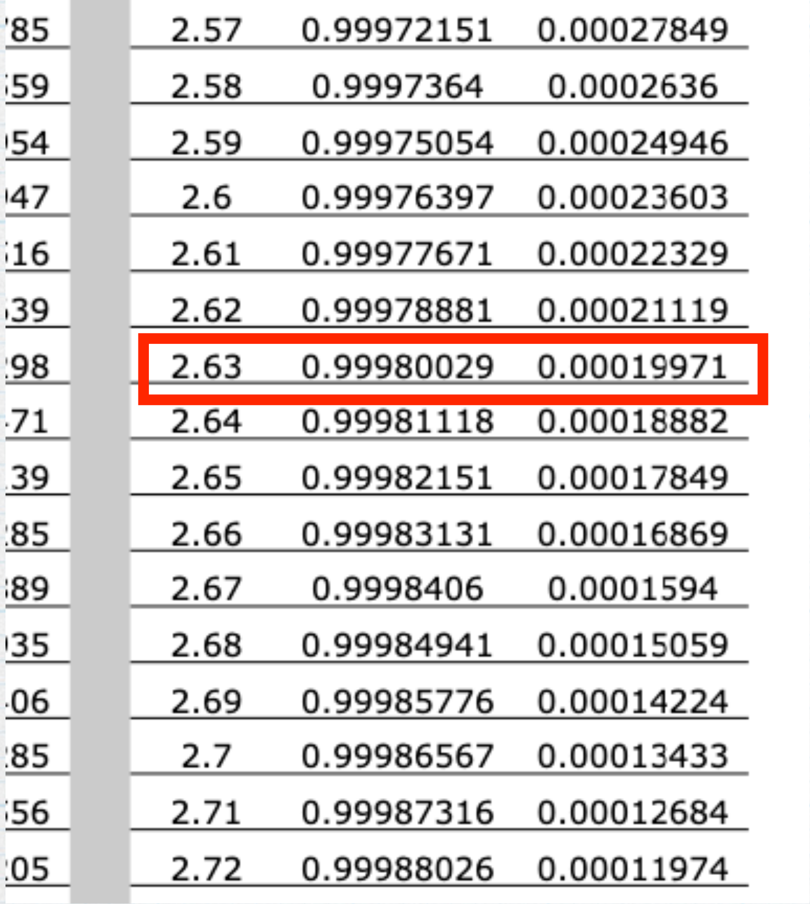
$$\mathrm{erf}(2.63) \approx 0.9998 \rightarrow \mathrm{erf}^{-1}(0.9998) \approx 2.63.$$

so that

$$x_j = 2\sqrt{1.67 \times 10^{-10}\,\mathrm{cm}^2}\,(2.63) = 6.8 \times 10^{-5}\,\mathrm{cm} = 0.68\,\mu\mathrm{m}$$

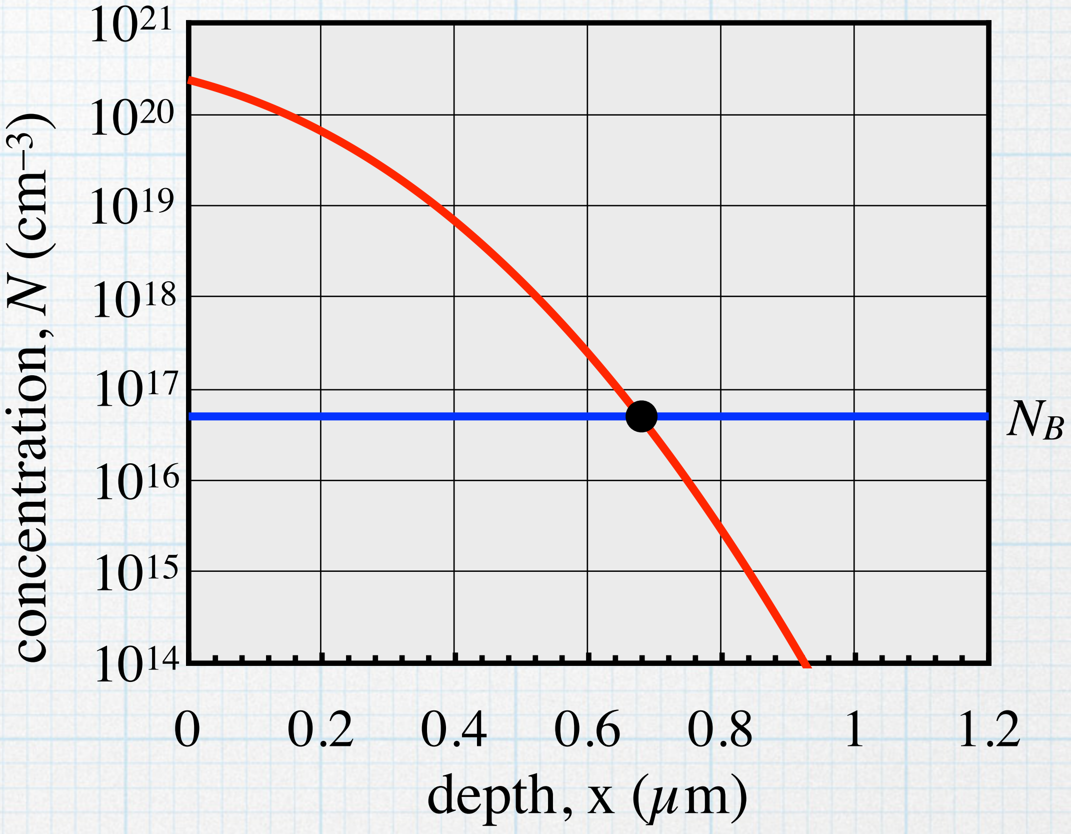
Example 1 (cont.)

Clip from the error-function table



85	2.57	0.99972151	0.00027849
59	2.58	0.9997364	0.0002636
54	2.59	0.99975054	0.00024946
47	2.6	0.99976397	0.00023603
16	2.61	0.99977671	0.00022329
39	2.62	0.99978881	0.00021119
98	2.63	0.99980029	0.00019971
71	2.64	0.99981118	0.00018882
39	2.65	0.99982151	0.00017849
85	2.66	0.99983131	0.00016869
89	2.67	0.9998406	0.0001594
35	2.68	0.99984941	0.00015059
06	2.69	0.99985776	0.00014224
85	2.7	0.99986567	0.00013433
56	2.71	0.99987316	0.00012684
05	2.72	0.99988026	0.00011974

Boron constant-source profile.



To finish, calculate the boron dose introduced into the silicon. For a constant-source diffusion:

$$Q = \frac{2N(0)}{\sqrt{\pi}}\sqrt{Dt}.$$

$$= \frac{2\left(2.4\times 10^{20}\,\mathrm{cm}^{-3}\right)}{\sqrt{\pi}}\sqrt{1.67\times 10^{-10}\,\mathrm{cm}^{2}} = 3.5\times 10^{15}\,\mathrm{cm}^{-2}$$

Example 2

A constant-dose arsenic diffusion is performed. The initial dose is $Q = 10^{14}$ cm^{-2}. The diffusion temperature is 1100°C and the diffusion time is 2 hr. The starting wafer has a *p*-type background doping of $N_B = 10^{17}$ cm^{-3}. Find the concentration of the arsenic at the surface and find the junction depth.

Again, we need to find *Dt* for the diffusion. From the diffusion data table:

$$D = D_o \exp\left(-\frac{E_A}{kT}\right)$$

$$= \left(9.17\,\frac{\text{cm}^2}{\text{s}}\right)\exp\left[-\frac{3.99\,\text{eV}}{\left(8.617\times10^{-5}\,\frac{\text{eV}}{\text{K}}\right)\left(1323\,\text{K}\right)}\right]$$

$$= 2.07\times10^{-14}\,\frac{\text{cm}^2}{\text{s}}$$

$$Dt = \left(2.07\times10^{-14}\,\frac{\text{cm}^2}{\text{s}}\right)\left(7200\,\text{s}\right) = 1.49\times10^{-10}\,\text{cm}^2$$

Example 2 (cont.)

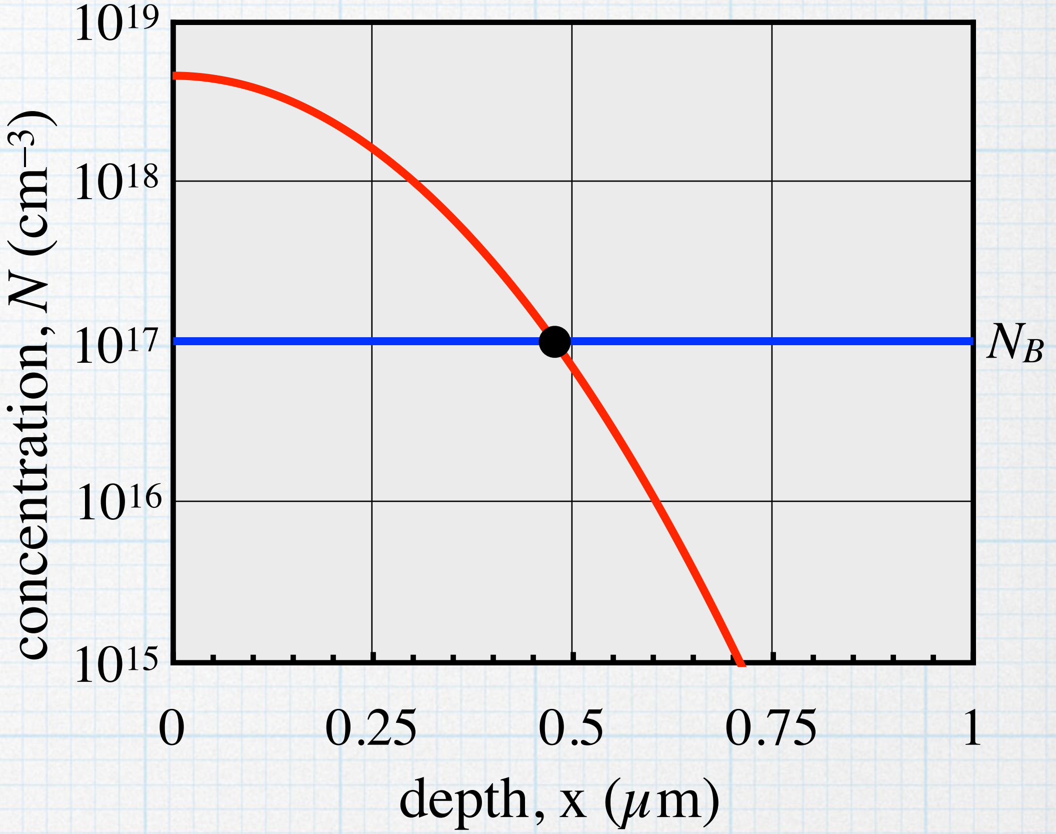
Find the surface concentration

$$N(0) = \frac{Q}{\sqrt{\pi D t}}$$

$$= \frac{10^{14}\,\text{cm}^{-2}}{\sqrt{\pi\left(1.49\times10^{-10}\,\text{cm}^2\right)}}$$

$$= 4.6\times10^{18}\,\text{cm}^{-3}.$$

arsenic constant-dose profile



Using the junction depth formula for Gaussian diffusions:

$$x_j = \sqrt{4Dt\cdot\ln\left[\frac{N(0)}{N_B}\right]}$$

$$= \sqrt{4\left(1.49\times10^{-10}\,\text{cm}^2\right)\cdot\ln\left(\frac{4.6\times10^{18}\,\text{cm}^{-3}}{10^{17}\,\text{cm}^{-3}}\right)}$$

$$= 4.8\times10^{-5}\,\text{cm} = 0.48\,\mu\text{m}$$

Example 3

In a two-step process, phosphorus is diffused into a *p*-type silicon wafer having $N_B = 10^{16}\ \text{cm}^{-3}$. For the deposition step, the temperature is 900°C and the time is 45 minutes. In the drive step, the temperature is 1100°C and the time is 60 minutes. Find the surface concentration and junction depth at the end of the two-step process.

Find $D_s t_s$ for the first step

$$D = D_o \exp\left(-\frac{E_A}{kT}\right)$$

$$D_s = \left(4.70\,\frac{\text{cm}^2}{\text{s}}\right)\exp\left[-\frac{3.68\,\text{eV}}{\left(8.617\times10^{-5}\,\frac{\text{eV}}{\text{K}}\right)\left(1173\,\text{K}\right)}\right]$$

$$= 7.25\times10^{-16}\,\frac{\text{cm}^2}{\text{s}}$$

$$D_s t_s = \left(7.25\times10^{-16}\,\frac{\text{cm}^2}{\text{s}}\right)\left(2700\,\text{s}\right) = 1.96\times10^{-12}\,\text{cm}^2$$

Example 3 (cont.)

The dose introduced during the first step is

$$Q = \frac{2N_s}{\sqrt{\pi}}\sqrt{D_s t_s}$$

N_s is the solid-solubility of phosphorus in silicon at 900°C. From the table, $N_s = 7\text{x}10^{20}\ \text{cm}^{-3}$, giving

$$Q = \frac{2\left(7\times 10^{20}\,\text{cm}^{-3}\right)}{\sqrt{\pi}}\sqrt{1.96\times 10^{-12}\,\text{cm}^2} = 1.11\times 10^{15}\,\text{cm}^{-2}$$

Next, calculate $D_d t_d$, for the drive step

$$D_d = \left(4.70\,\frac{\text{cm}^2}{\text{s}}\right)\exp\left[-\frac{3.68\,\text{eV}}{\left(8.617\times 10^{-5}\,\frac{\text{eV}}{\text{K}}\right)\left(1373\,\text{K}\right)}\right]$$

$$= 1.46\times 10^{-13}\,\frac{\text{cm}^2}{\text{s}}$$

$$D_d t_d = \left(1.46\times 10^{-13}\,\frac{\text{cm}^2}{\text{s}}\right)\left(3600\,\text{s}\right) = 5.25\times 10^{-10}\,\text{cm}^2$$

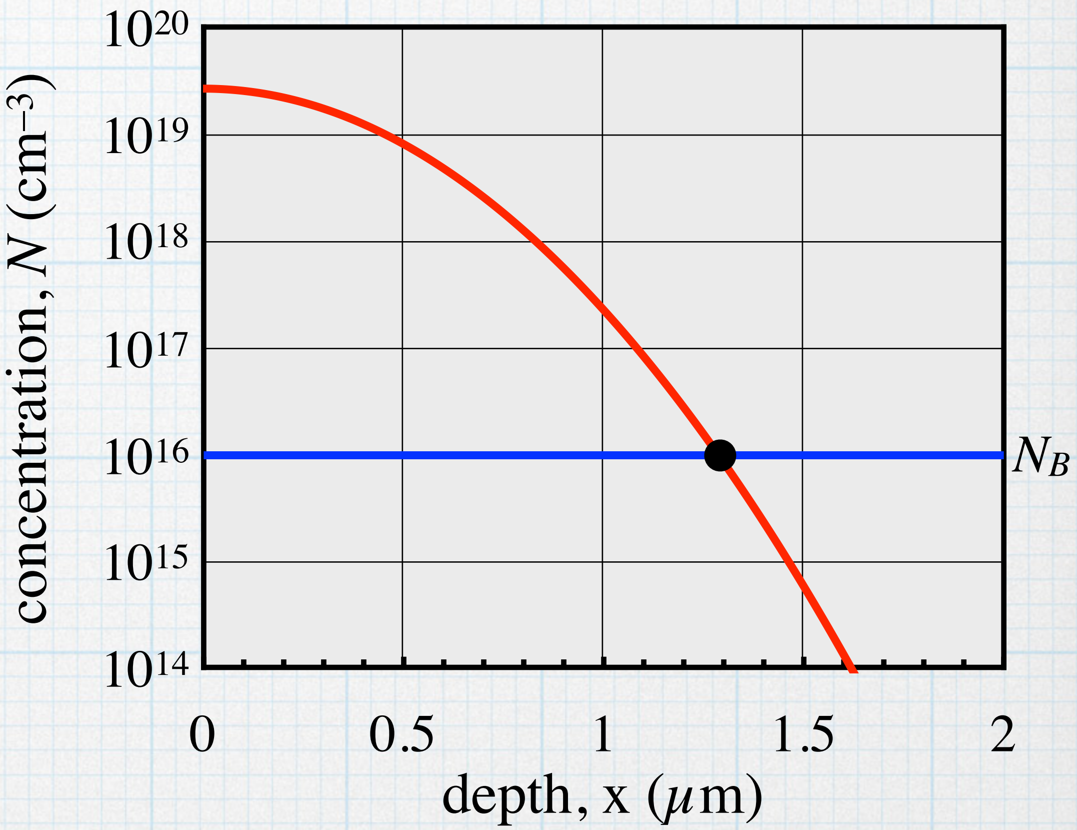
Example 3 (cont.)

Now, we can find the surface concentration:

$$N(0) = \frac{Q}{\sqrt{\pi\left(D_d t_d\right)}}$$

$$= \frac{1.1 \times 10^{15}\,\mathrm{cm}^{-2}}{\sqrt{\pi\left(5.25 \times 10^{-10}\,\mathrm{cm}^{-2}\right)}}$$

$$= 2.73 \times 10^{19}\,\mathrm{cm}^{-3}$$



And the junction depth:

$$x_j = \sqrt{4\left(D_d t_d\right)\ln\left[\frac{N(0)}{N_B}\right]}$$

$$= \sqrt{4\left(5.25 \times 10^{-10}\,\mathrm{cm}^2\right)\cdot\ln\left[\frac{2.71 \times 10^{19}\,\mathrm{cm}^{-3}}{10^{16}\,\mathrm{cm}^{-3}}\right]}$$

$$= 1.29 \times 10^{-4}\,\mathrm{cm} = 1.29\,\mu\mathrm{m}$$

Example 4

Design a constant-source diffusion of phosphorus into *p*-type silicon that produces a dose of $Q = 5\text{x}10^{15}\ \text{cm}^{-3}$ Also, calculate the resulting junction depth. The background *p*-type doping in the silicon is $N_B = 1\text{x}10^{17}\ \text{cm}^{-3}$. The surface concentration will be determined by the solid solubility limit of phosphorus at the diffusion temperature.

There aren't too many degrees of freedom with a constant-source diffusion. The general equation is:

$$N(x,t) = N(0)\,\text{erfc}\left(\frac{x}{2\sqrt{Dt}}\right) = N_s\left[1 - \text{erf}\left(\frac{x}{2\sqrt{Dt}}\right)\right]$$

Because the surface concentration will be equal to the solid solubility limit, we might start by choosing the diffusion temperature. Let's pick 1000°C. At that temp, the SSL for phosphorus is $N_s = 1.2 \times 10^{21}\ \text{cm}^{-3}$. The dose for a constant-source diffusion is given by

$$Q = \frac{2N(0)}{\sqrt{\pi}}\sqrt{Dt}.$$

Example 4 (cont.)

Then we can calculate the required Dt for the diffusion

$$Dt = \pi\left[\frac{Q}{2N_s}\right]^2 = \pi\left[\frac{5\times10^{15}\,\mathrm{cm^{-2}}}{2\left(1.2\times10^{21}\,\mathrm{cm^{-3}}\right)}\right]^2 = 1.36\times10^{-11}\,\mathrm{cm^2}$$

At T = 1000°C = 1273 K, the diffusion coefficient for phosphorus is

$$D = \left(4.7\,\frac{\mathrm{cm^2}}{\mathrm{s}}\right)\exp\left[-\frac{3.68\,\mathrm{eV}}{\left(8.617\times10^{-5}\,\frac{\mathrm{eV}}{\mathrm{K}}\right)(1273\,\mathrm{K})}\right] = 1.27\times10^{-14}\,\frac{\mathrm{cm^2}}{\mathrm{s}}$$

The diffusion time is then

$$t = \frac{(Dt)}{D} = \frac{1.36\times10^{-11}\,\mathrm{cm^2}}{1.27\times10^{-14}\,\frac{\mathrm{cm^2}}{\mathrm{s}}} = 1071\,\mathrm{s} = 17.8\,\mathrm{min}$$

Example 4 (cont.)

Finally, we can calculate the junction depth:

$$x_j = 2\sqrt{Dt}\left[\mathrm{erf}^{-1}\left(1 - \frac{N_B}{N_s}\right)\right]$$

With $1 - \dfrac{N_B}{N_s} = 1 - \dfrac{5 \times 10^{16}\,\mathrm{cm}^{-3}}{2.4 \times 10^{24}\,\mathrm{cm}^{-3}} = 0.9999167.$

Using the error function table again — or Wolfram-Alpha — we see that

$$\mathrm{erf}(2.782) \approx 0.9999166 \rightarrow \mathrm{erf}^{-1}(0.9999166) \approx 2.782.$$

Close enough.

So

$$x_j = 2\sqrt{1.36 \times 10^{-11}\,\mathrm{cm}^2}\,(2.782) = 2.05 \times 10^{-5}\,\mathrm{cm} = 0.205\,\mu\mathrm{m}$$

Example 5

Design a constant-dose diffusion of arsenic into p-type silicon that gives a final surface concentration of $N(0) = 5\text{x}10^{18}\ \text{cm}^{-3}$ and a junction depth of $x_j = 1\ \mu\text{m}$. The background p-type doping in the silicon is $N_B = 5\text{x}10^{16}\ \text{cm}^{-3}$. Find the required Q and Dt. Also, pick times and temperatures for Dt.

Start with the constant-dose equation

$$N(x,t) = \frac{Q}{\sqrt{\pi Dt}} \exp\left(-\frac{x^2}{4Dt} \right) = N(0) \exp\left(-\frac{x^2}{4Dt} \right)$$

We can use given requirement for junction depth and surface concentration to find Dt

$$N_B = N(0) \exp\left(-\frac{x_j^2}{4Dt} \right)$$

$$Dt = \frac{x_j^2}{4 \ln\left[\frac{N(0)}{N_B}\right]} = \frac{\left(10^{-4}\ \text{cm}\right)^2}{4 \ln\left[\frac{5\times10^{18}\ \text{cm}^{-3}}{5\times10^{16}\ \text{cm}^{-3}}\right]} = 5.43\times10^{-10}\ \text{cm}^2$$

Example 5 (cont.)

Now we can find the required value of Q:

$$Q = N(0)\sqrt{\pi D t}$$

$$= \left(5\times10^{18}\,\mathrm{cm}^{-3}\right)\sqrt{\pi\left(5.43\times10^{-10}\,\mathrm{cm}^{2}\right)} = 2.07\times10^{14}\,\mathrm{cm}^{-2}$$

As before, the selection of time and temperature is somewhat arbitrary, since we need only specify the final Dt product. This time, let's choose a diffusion time of $t = 3$ hours = 10,800 s. Then

$$D = \frac{Dt}{t} = \frac{5.43\times10^{-10}\,\mathrm{cm}^{2}}{10800s} = 5.03\times10^{-14}\,\frac{\mathrm{cm}^{2}}{\mathrm{s}}$$

For arsenic

$$D = \left(9.17\,\frac{\mathrm{cm}^{2}}{\mathrm{s}}\right)\exp\left(-\frac{3.99\,\mathrm{eV}}{kT}\right)$$

$$kT = \frac{-3.99\,\mathrm{eV}}{\ln\left(\frac{5.03\times10^{-14}}{9.17}\right)} = 0.12151\,\mathrm{eV} \;\;\rightarrow\;\; T = 1410\,\mathrm{K} = 1137^{\circ}\,\mathrm{C}$$

Example 6

Design a two-step diffusion of boron into *n*-type silicon that will result in a surface concentration of $N(0) = 5\text{x}10^{17}$ cm^{-3} and a junction depth of $x_j = 2\ \mu$m. The background *n*-type doping in the silicon is $N_B = 1\text{x}10^{15}$ cm^{-3}. Find the two different different Dt values and then specify times and temperature for each.

This will be similar to Example 5. Start by using the final surface connection and junction requirements depth to find D_2t_2:

$$N(x,t) = \frac{Q}{\sqrt{\pi D_d t_d}} \exp\left(-\frac{x^2}{4D_d t_d}\right) = N(0) \exp\left(-\frac{x^2}{4D_d t_d}\right)$$

$$N_B = N(0) \exp\left(-\frac{x_j^2}{4D_d t_d}\right)$$

$$\left(D_d t_d\right) = \frac{x_j^2}{4 \ln\left[\frac{N(0)}{N_B}\right]} = \frac{\left(2 \times 10^{-4}\,\text{cm}\right)^2}{4 \ln\left[\frac{5 \times 10^{17}\,\text{cm}^{-3}}{10^{15}\,\text{cm}^{-3}}\right]} = 1.61 \times 10^{-9}\,\text{cm}^{-3}$$

Example 6 (cont.)

Then we can use the surface concentration requirement to find $D_s t_s$.

$$N(0) = \frac{2N_s}{\pi}\sqrt{\frac{D_s t_s}{D_d t_d}}$$

where N_s is the solid solubility limit of boron at the temperature used for the deposition step. We need to choose that temperature now —let's use 825°C. From the solid-solubility table, we can interpolate between 800°C and 900°C to get $N_s = 8.25 \times 10^{19}\,\text{cm}^{-3}$. Then

$$D_s t_s = \left[\frac{\pi N(0)}{2N_s}\right]^2 D_d t_d$$

$$= \left[\frac{\pi\left(5 \times 10^{17}\,\text{cm}^{-3}\right)}{2\left(8.25 \times 10^{19}\,\text{cm}^{-3}\right)}\right]^2 \left(1.61 \times 10^{-9}\,\text{cm}^2\right)$$

$$= 1.46 \times 10^{-13}\,\text{cm}^2$$

Example 6 (cont.)

Having chosen 825°C for the deposition, the diffusion coefficient will be:

$$D_s = \left(1.0\,\frac{\text{cm}^2}{\text{s}}\right)\exp\left[-\frac{3.50\,\text{eV}}{\left(8.617\times10^{-5}\,\frac{\text{eV}}{\text{K}}\right)(1098\,\text{K})}\right]$$

$$= 8.60\times10^{-17}\,\frac{\text{cm}^2}{\text{s}}$$

The deposition time is then:

$$t_s = \frac{(D_s t_s)}{D_1} = \frac{1.46\times10^{-13}\,\text{cm}^2}{8.60\times10^{-17}\,\frac{\text{cm}^2}{\text{s}}} = 1698\,\text{s} = 28.3\,\text{min}$$

To finish, we need time and temperature for the drive step to get the prescribed $D_d t_d$. We will choose the drive temperature to be $T = 1100°\text{C} = 1373$ K. The corresponding boron diffusion coefficient is then $D_d = 1.42 \times 10^{-13}$ cm²/s, and the required drive time would be:

$$t_2 = \frac{(D_d t_d)}{D_d} = \frac{1.61\times10^{-9}\,\text{cm}^2}{1.42\times10^{-13}\,\frac{\text{cm}^2}{\text{s}}} = 11{,}338\,\text{s} = 3.15\,\text{hr}.$$

Example 7

In a two-step process, boron was diffused into an *n*-type silicon wafer having $N_B = 10^{15}$ cm^{-3}. In the deposition step, the temperature was 850°C and the time was 45 minutes. The drive was done in multiple steps spread over a few days.

- 4 hours at 1125°C (1398 K)
- 12 hours at 800°C (1023 K)
- 10 hours at 1050°C (1323 K)
- 12 hours at 800°C (1023 K)
- 6 hours at 1100°C (1373 K)

Find the surface concentration and junction depth at the end of the two-step process.

This will be similar to Example 3. First, find $D_s t_s$ for the sourcing step

$$D_s t_s = \left(1.0\,\frac{\text{cm}^2}{\text{s}}\right)\exp\left[-\frac{3.50\,\text{eV}}{\left(8.617\times10^{-5}\,\frac{\text{eV}}{\text{K}}\right)\left(1123\,\text{K}\right)}\right]\left(2700\,\text{s}\right)$$

$$= 5.29\times10^{-13}\,\text{cm}^2$$

Example 7 (cont.)

The solid-solubility of boron in silicon at 850°C is $N_s = 9.5\text{x}10^{19}\ \text{cm}^{-3}$. Then the dose is

$$Q = \frac{2N_s}{\sqrt{\pi}}\sqrt{D_s t_s} = \frac{2\left(9.5\times10^{19}\,\text{cm}^{-3}\right)}{\sqrt{\pi}}\sqrt{5.29\times10^{-13}\,\text{cm}^2} = 7.80\times10^{13}\,\text{cm}^{-2}$$

Since the drive step comes with various different times and temperatures, we need to add up the *Dt*-productss from the individual step to obtain the composite value, as described in the notes.

$$D_d t_d = \sum_{n=1}^{5} D_n t_n$$

For the first portion (T_1 = 1125°C and t_1 = 4 hr = 14,400 s)

$$D_1 t_1 = \left(1.0\,\frac{\text{cm}^2}{\text{s}}\right)\exp\left[-\frac{3.50\,\text{eV}}{\left(8.617\times10^{-5}\,\frac{\text{eV}}{\text{K}}\right)\left(1123\,\text{K}\right)}\right]\left(2700\,\text{s}\right)$$

$$= 3.47\times10^{-9}\,\text{cm}^2$$

Example 7 (cont.)

And similar for each of the remaining steps:

- T_2 = 1125°C and t_2 = 12 hr : $D_2 t_2 = 1.57\text{x}10^{-12}$ cm^2
- T_3 = 1050°C and t_3 = 10 hr : $D_3 t_3 = 1.67\text{x}10^{-9}$ cm^2
- Same at step 2 : $D_4 t_4 = 1.57\text{x}10^{-12}$ cm^2
- T_5 = 1100°C and t_5 = 6 hr : $D_5 t_5 = 3.06\text{x}10^{-9}$ cm^2

Adding the five individual *Dt* values together, we obtain the cumulative *Dt* for the drive:

$$D_d t_d = \sum_{n=1}^{5} D_n t_n = 8.20 \times 10^{-9}\,\text{cm}^2$$

We note that the second and fourth steps had negligible contribution to the total — at the lower temperature, the diffusion coefficient is so small that even long times can’t make *Dt* appreciable.

Example 7 (cont.)

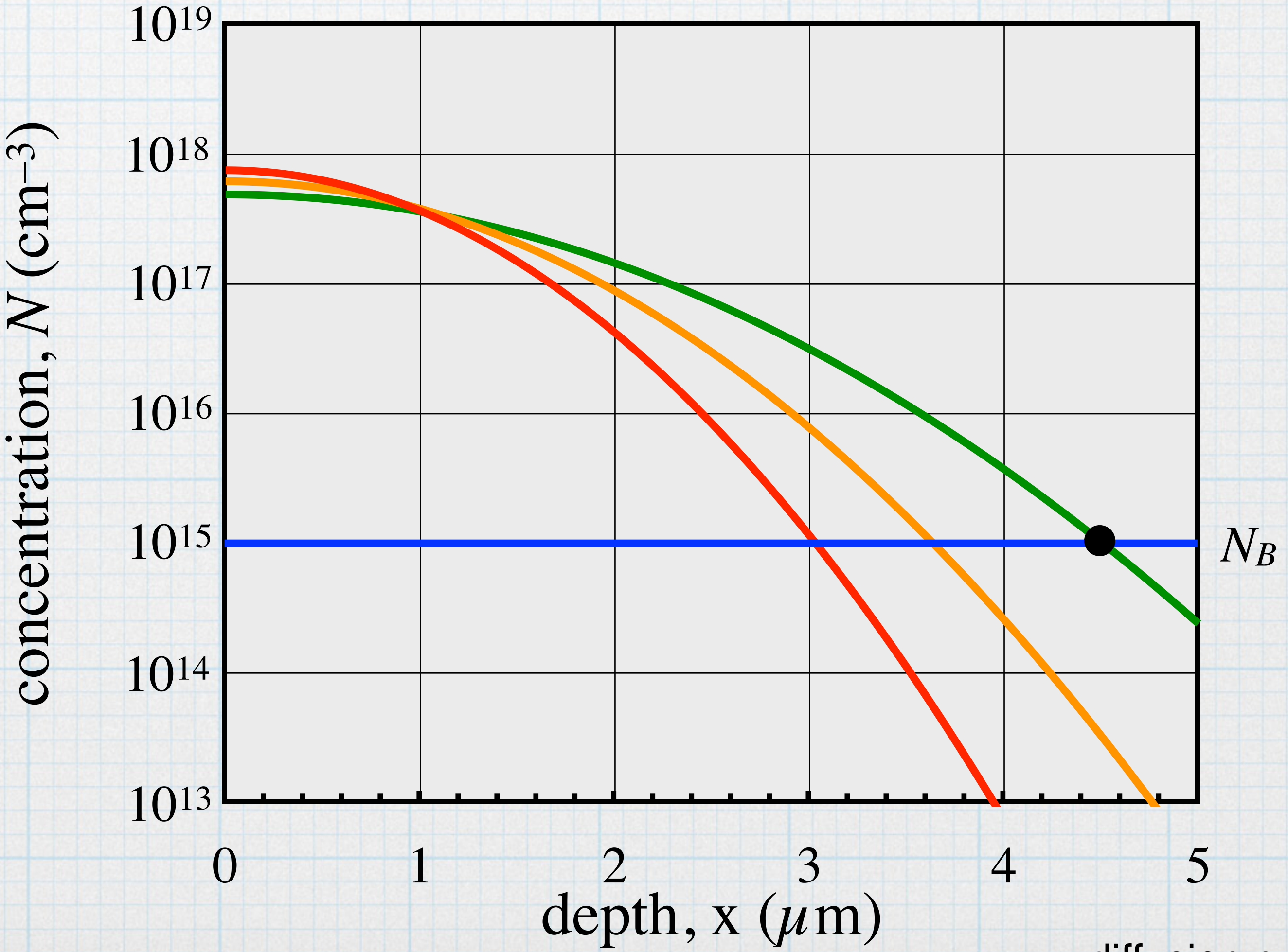
The rest is similar to earlier examples. Find the surface concentration:

$$N(0) = \frac{Q}{\sqrt{\pi\left(D_d t_d\right)}} = 4.86 \times 10^{17}\,\mathrm{cm}^{-3}$$

And the junction depth:

$$x_j = \sqrt{4\left(D_d t_d\right)\ln\left[\frac{N(0)}{N_B}\right]} = 4.50 \times 10^{-4}\,\mathrm{cm} = 4.50\,\mu\mathrm{m}$$

We can see the progressive effect on the boron profile after the 1st (red), 3rd (orange), and 5th (green) steps. (The 2nd and 4th steps had virtually no effect on the profile.)



Example 8

In a process that is part of making some type of transistor, two constant-source (Gaussian) diffusions are perfumed into an *n*-type silicon wafer that has background doping of $N_B = 5\text{x}10^{14}\ \text{cm}^{-3}$.

- The first diffusion is boron with parameters: $Q_1 = 10^{14}\ \text{cm}^{-2}$, $T_1 = 1050°\text{C}$, and $t_1 = 2\ \text{hr}$.
- The second diffusion is phosphorus with parameters: $Q_2 = 10^{15}\ \text{cm}^{-2}$, $T_2 = 1025°\text{C}$, and $t_2 = 60\ \text{min}$.

Two junctions will be formed. Find the two junction depths.

The boron diffusion creates a *p*-type layer at the surface of the wafer. The D_1t_1 for the boron step is

$$D_1t_1 = \left(1.0\,\frac{\text{cm}^2}{\text{s}}\right)\exp\left[-\frac{3.50\,\text{eV}}{\left(8.617\times10^{-5}\,\frac{\text{eV}}{\text{K}}\right)\left(1323\,\text{K}\right)}\right]\left(7200\,\text{s}\right)$$

$$= 3.34\times10^{-10}\,\text{cm}^2$$

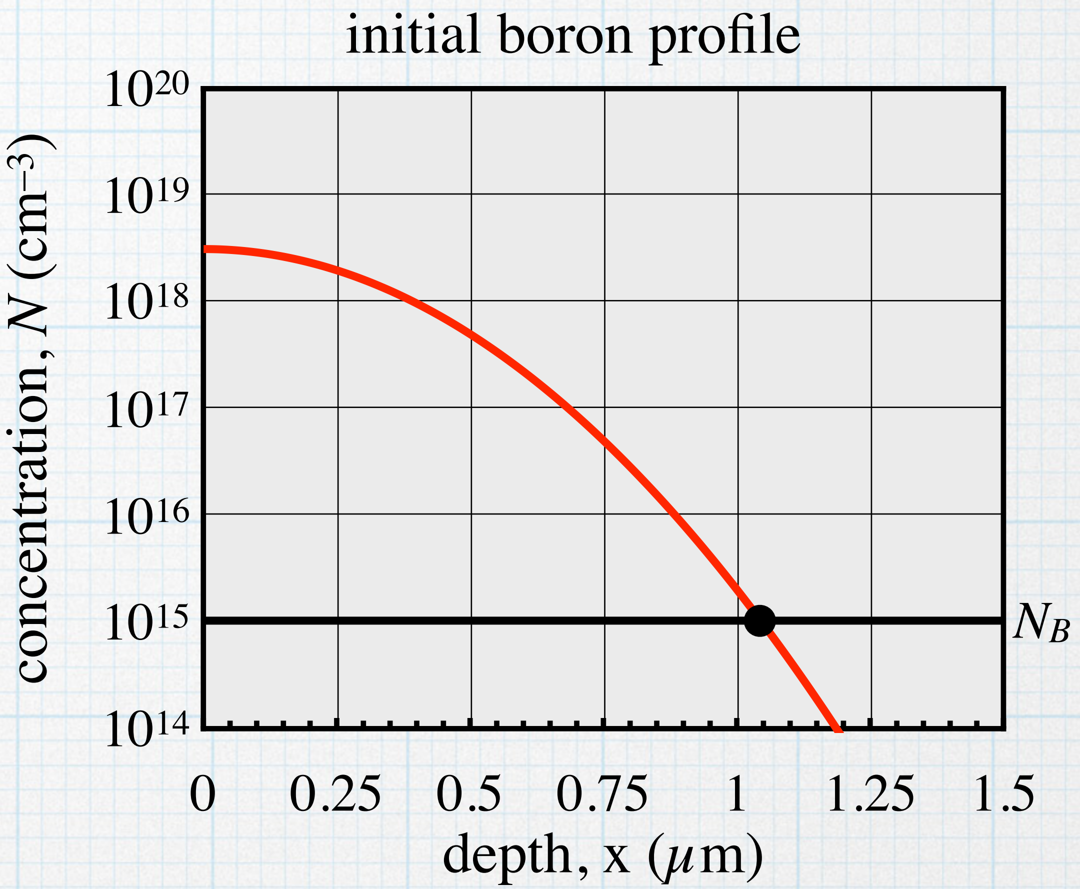
Example 8 (cont.)

The resulting surface concentration is

$$N_1(0) = \frac{Q_1}{\sqrt{\pi D_1 t_1}}$$

$$= \frac{10^{14}\,\mathrm{cm}^{-2}}{\sqrt{\pi\left(3.34\times10^{-10}\,\mathrm{cm}^2\right)}}$$

$$= 3.09\times10^{18}\,\mathrm{cm}^{-3}.$$



The junction depth between the p-type boron profile and the n-type substrate is

$$x_{j1} = \sqrt{4D_1t_1\cdot\ln\left[\frac{N_1(0)}{N_B}\right]} = 1.04\times10^{-4}\,\mathrm{cm} = 1.04\,\mu\mathrm{m}$$

Example 8 (cont.)

The phosphorus diffusion compensates the a *p*-type boron near the surface, creating an *n-p-n* "sandwich". The D_2t_2 for the phos. diffusion is

$$D_2t_2 = \left(4.7\,\frac{\text{cm}^2}{\text{s}}\right)\exp\left[-\frac{3.68\,\text{eV}}{\left(8.617\times10^{-5}\,\frac{\text{eV}}{\text{K}}\right)\left(1298\,\text{K}\right)}\right]\left(3600\,\text{s}\right)$$

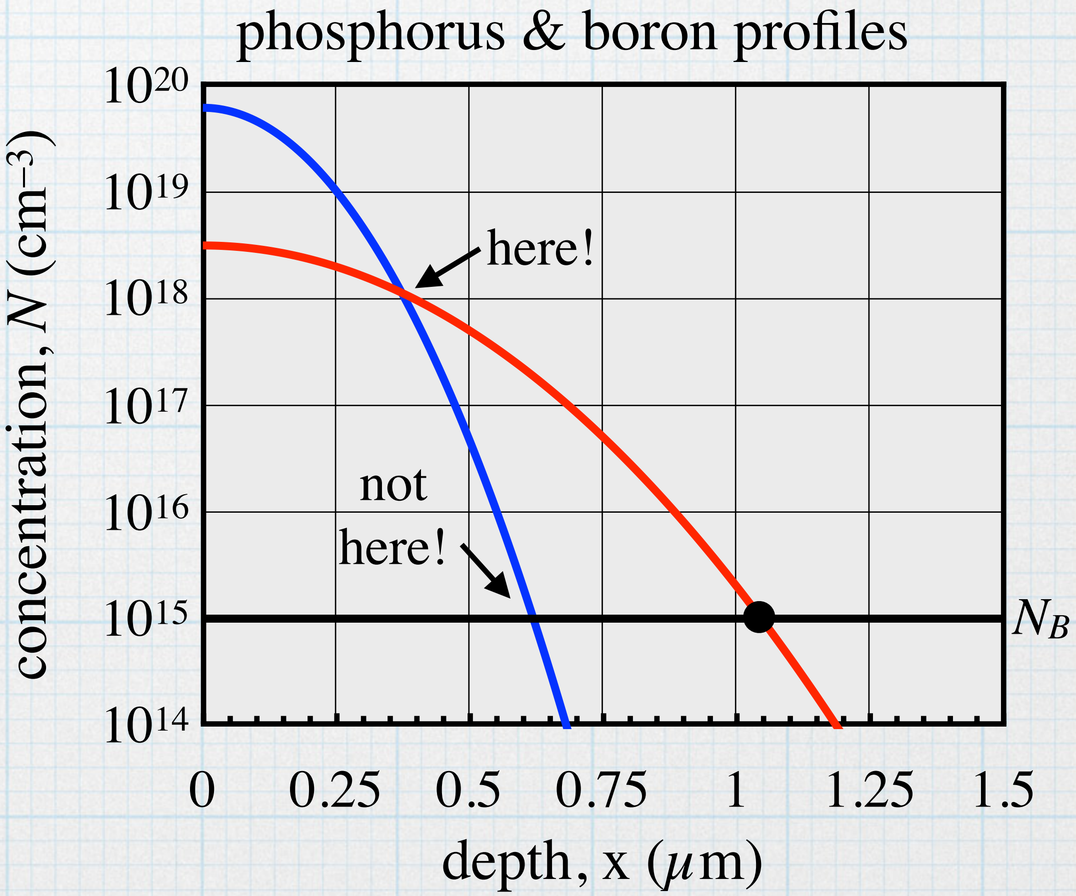
$$= 8.70\times10^{-11}\,\text{cm}^2.$$

The resulting surface concentration is

$$N_2(0) = \frac{Q_2}{\sqrt{\pi\left(D_2t_2\right)}}$$

$$= \frac{10^{15}\,\text{cm}^{-2}}{\sqrt{\pi\left(8.70\times10^{-11}\,\text{cm}^2\right)}}$$

$$= 6.05\times10^{19}\,\text{cm}^{-3}.$$



Example 8 (cont.)

However, the junction is *not* between $N_2(x)$ and the substrate (both are n-type). The junction is between the n-type phosphorus diffusion and the p-type boron diffusion. Therefore, we must solve the relationship:

$$N_2\left(x_{j2}\right) = N_1\left(x_{j2}\right).$$

This seems bad, but really it's not too awful — both profiles are Gaussian:

$$N_2(0)\exp\left(-\frac{x_{j2}}{D_2 t_2}\right) = N_1(0)\exp\left(-\frac{x_{j2}}{D_1 t_1}\right).$$

After some manipulation, we arrive at an expression for x_{j2}:

$$x_{j2} = \sqrt{4\left[\frac{D_1 t_1 D_2 t_2}{D_1 t_1 - D_2 t_2}\right] \cdot \ln\left(\frac{N_2(0)}{N_1(0)}\right)}.$$

Inserting values and calculating, the second junction is at

$$x_{j2} = \sqrt{4\left[\frac{\left(3.34\times10^{-10}\,\mathrm{cm}^2\right)\left(8.70\times10^{-11}\,\mathrm{cm}^2\right)}{3.34\times10^{-10}\,\mathrm{cm}^2 - 8.70\times10^{-11}\,\mathrm{cm}^2}\right] \cdot \ln\left(\frac{6.05\times10^{19}\,\mathrm{cm}^{-3}}{3.09\times10^{18}\,\mathrm{cm}^{-3}}\right)}$$

$$= 3.74\times10^{-5}\,\mathrm{cm} = 0.374\,\mu\mathrm{m}.$$

Example 8 (cont.)

But wait! From the notes and from Example 7, we know that the boron will diffuse more during the time that the wafer is hot for the phosphorus diffusion. We should account for it by adding "extra" *Dt* to the boron diffusion

$$D_1' t_1' = \left(1.0\,\frac{\mathrm{cm}^2}{\mathrm{s}}\right)\exp\left[-\frac{3.50\,\mathrm{eV}}{\left(8.617\times10^{-5}\,\frac{\mathrm{eV}}{\mathrm{K}}\right)\left(1298\,\mathrm{K}\right)}\right]\left(3600\,\mathrm{s}\right)$$

$$= 9.25\times10^{-11}\,\mathrm{cm}^2.$$

Then the corrected *Dt* is a bit bigger:

$$D_1 t_1 = 3.34\times10^{-10}\,\mathrm{cm}^2 + 9.25\times10^{-11}\,\mathrm{cm}^2 = 4.27\times10^{-10}\,\mathrm{cm}^2.$$

The new value of $D_1 t_1$ leads to the following changes:

$$N_1(0) = 2.73\times10^{18}\,\mathrm{cm}^{-3}$$

$$x_{j1} = 1.16\,\mu\mathrm{m}$$

$$x_{j2} = 0.368\,\mu\mathrm{m}$$

The changes are marginal — the boron-substrate junction is slightly deeper and the upper junction is ever-so-slightly shallower.

Example 8 (cont.)

Boron and phosphorus final doping profiles. The dotted red line indicates the initial boron profile, and the solid red line shows how the boron shifted during phosphorus diffusion.