The electron sheet concentration under the gate is

$$qn_s = C_{ox}\left[V_G - \phi\left(y\right) - V_T\right]$$

where $\phi(y)$ is the local electrostatic potential in the semiconductor just under the gate.

If the drain voltage is also at ground potential, then all parts of the semiconductor (source, body, and drain) are all at $\phi = 0$ and

$$qn_s = C_{ox}\left[V_G - V_T\right]$$

Of course, with $V_{DS} = 0$, no current will flow. If we increase V_{DS} slightly, say just a few millivolts, we can assume that the electron sheet concentration is still essentially uniform, but a drift current will begin to flow from drain to source. (Electrons are moving from source to drain.)

Treating the channel inversion layer like a resistor,

$$i_D = \frac{V_{DS}}{R_{channel}}$$

$$R_{channel} = \frac{\rho L}{A} = \frac{L}{q\mu_n n_{inv} W t_{inv}}$$

where n_{inv} is the electron concentration (number per unit volume) of the inversion layer and t_{inv} is the thickness of the inversion layer. Unfortunately, we don't either of these quantities. However, we do know

$$n_s = n_{inv} \cdot t_{inv}$$

which is the electron sheet concentration of inversion layer. Putting everything together.

$$i_D = q\mu_n n_s \frac{W}{L} V_{DS}$$

$$= \mu_n C_{ox} \left[V_G - V_T\right] \frac{W}{L} V_{DS}$$

However, as V_{DS} increases, the potential along the channel must change correspondingly:

source: $\phi(0) = 0$

drain: $\phi(L) = V_{DS}$ with continuous variation in between.

So we have to look smaller. At any point along the channel,

$$J_n = \sigma \mathcal{E}$$

$$\frac{i_D}{W t_{inv}} = \left[q \mu_n n_{inv} \right] \left[-\frac{d\phi}{dy} \right]$$

$$i_D dy = -\left[q \mu_n n_{inv} W \right] d\phi$$

$$i_D dy = -\left[q \mu_n n_{inv} t_{inv} W \right] d\phi$$

$$i_D dy = -\left[\mu_n C_{ox} \left(V_G - \phi\left(y\right) - V_T \right) W \right] d\phi$$

The current must be continuous (Kirchoff's current law), and so i_D is a constant on the LHS of the equation. Then integrating both sides:

$$i_D \int_0^L dy = -\mu_n C_{ox} W \int_0^{V_D} \left[V_G - \phi\left(y\right) - V_T\right] d\phi$$

$$i_D L = +\frac{\mu_n C_{ox} W}{2} \left[V_G - V_T - \phi\right]^2 \Big|_0^{V_D}$$

$$= \frac{1}{2}\mu_n C_{ox} \frac{W}{L} \left[\left(V_G - V_T - V_{DS}\right)^2 - \left(V_G - V_T\right)^2\right]$$

$$= \frac{1}{2}\mu_n C_{ox} \frac{W}{L} \left[\left(V_G - V_T\right) V_{DS} - V_{DS}^2\right]$$

$$= K_n \left[\left(V_G - V_T\right) V_{DS} - V_{DS}^2\right] \qquad K_n = \frac{1}{2}\mu_n C_{ox} \frac{W}{L}$$